

**POST GRADUATE DIPLOMA IN
APPLIED STATISTICS (PGDAST)**

Term-End Examination

June, 2022

MST-003 : PROBABILITY THEORY

Time : 3 hours

Maximum Marks : 50

Note :

- (i) Question no. 1 is **compulsory**.
- (ii) Attempt any **four** questions from the remaining questions no. 2 to 7.
- (iii) Use of scientific (non-programmable) calculator is allowed.
- (iv) Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.
- (v) Symbols have their usual meanings.

1. State whether the following statements are *True* or *False*. Give reasons in support of your answers. $5 \times 2 = 10$

- (a) Two events are said to be independent if $P(A \cap B) = P(A) + P(B)$.
- (b) If the probability distribution of X is

X	- 2	- 1	0	1	2
p(x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

then F(0) will be $\frac{2}{3}$.

- (c) If the probability of hitting a target in any attempt is 0.6, then to find the probability that it would be hit on the fifth attempt, the suitable distribution will be exponential.
- (d) If X_1 and X_2 are independent variates such that $X_1 \sim N(0, 9)$ and $X_2 \sim N(2, 36)$, then SD of $X_2 - X_1$ will be $\sqrt{27}$.
- (e) If X follows uniform distribution $U(0, 10)$, then $P(X < 2)$ will be $\frac{1}{5}$.

2. (a) An electronic equipment has two essential parts, A and B. In the past, part A failed 30% of the times, part B failed 20% of the times and both the parts failed simultaneously 5% of the times. Assume that both the parts must be in operative mode for the equipment to perform its intended function. What is the probability that the equipment will function ? 4

(b) For two events A and B, given that $P(A) = 0.4$, $P(B) = p$ and $P(A \cup B) = 0.6$.

- (i) Find p so that A and B are independent events.
- (ii) For what value of p are A and B mutually exclusive ? 4

(c) Write the classical definition of probability. 2

3. (a) In a college fete, there is a stall, where on buying a ticket, a person is allowed one throw of two dice. If this gives a double six, 10 times of the ticket money is refunded. If one six turns up, double of the ticket money is refunded and in other cases nothing is refunded. Will it be profitable to run such a stall ? What is the expectation of the player ? 6

- (b) Two discrete random variables X and Y have the following joint probabilities :

$$P[X = 0, Y = 0] = \frac{2}{9}, \quad P[X = 0, Y = 1] = \frac{1}{9},$$

$$P[X = 1, Y = 0] = \frac{1}{9} \quad \text{and} \quad P[X = 1, Y = 1] = \frac{5}{9}.$$

Examine whether X and Y are independent. 4

4. (a) An item is produced by a machine in large numbers. The machine is known to produce 20% defectives. A quality control engineer is testing the items randomly. What is the probability that at least 3 items are examined in order to get 2 defectives ? 3

- (b) The average number of customers who arrive at a counter of a certain bank per minute is 2. Find the probability that during a given minute

- (i) no customer arrives.
(ii) three or more customers arrive. 3

- (c) Obtain the mean and SD of a binomial distribution for which
 $P(X = 3) = 16P[X = 7]$ and $n = 10$. 3
- (d) Define discrete uniform distribution. 1
5. (a) If the marks of assignment of MST-002 follows normal distribution with mean 79 and SD 5, then in a batch of 200 learners, how many of them
- (i) did not receive marks between 75 and 82 ?
- (ii) received less than 75 ?
- (iii) received more than 90 ? 8
- (b) In a city, buses are scheduled every 20 minutes at a certain stop. A person comes to the stop at a random time and waits for the bus. If the waiting time follows uniform distribution over the interval $(0, 20)$, then find the probability that she/he has to wait at least 15 minutes for the bus. 2
6. (a) Suppose that on an average, one customer per minute arrives at a shop. What is the probability that the shopkeeper will wait more than 3 minutes before the first two customers arrive ? Assume that waiting time follows :
- (i) Gamma distribution. 4
- (ii) Exponential distribution. 2

- (b) An unbiased coin is cast until a head appears. What is the probability that it must be cast more than 3 times ?

4

7. (a) A company has two plants to manufacture LED bulbs. Plant I manufactures 70% of the LED bulbs and Plant II manufactures 30%. At Plant I, 80% of the LED bulbs produced are of standard quality and at Plant II, 90% of the LED bulbs produced are of standard quality. An LED bulb is picked up at random and is found to be of standard quality. What is the chance that it has come from Plant II ?

5

- (b) The joint density function of random variables X and Y is given by

$$f(x, y) = \begin{cases} x + y; & 0 \leq x < 1 \text{ and } 0 \leq y < 1 \\ 0; & \text{otherwise.} \end{cases}$$

- (i) Find the marginal pdfs of X and Y.
(ii) Are X and Y independent ?
(iii) Find the conditional density function of Y given X.

5
