No. of Printed Pages : 6 **MMTE-005**

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. Sc. (MACS)]

Term-End Examination

June, 2022

MMTE-005 : CODING THEORY

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Answer any **four** questions from question nos. **1** to **5**.*

*(ii) Question No. **6** is compulsory.*

(iii) All questions carry equal marks.

*(iv) Use of calculator is **not** allowed.*

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1. (a) Consider the [7, 4] binary code with the following generator matrix : 4

$$\left[\begin{array}{cccc}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{array}\right]\left[\begin{array}{ccc}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\\ 1 & 1 & 1\end{array}\right]$$

(i) Write the parity check matrix.

(ii) Find four information sets in the above code.

(iii) Find one set of 4 co-ordinates that do not form an information set.

(b) (i) Find the dimension and minimum weight of the Reed-Muller code R (2, 4).

(ii) Find the generator matrix of the Reed-Muller code R (3, 4).

(iii) The parity check matrix of [15, 11] binary Hamming code is given below :

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Assume that the received vector is (0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1). Find the correct decoded message. 6

2. (a) (i) Show that the polynomial $f(x) = x^3 + x + 1$ is irreducible in $\mathbf{F}_2 [x]$.

(ii) Let $\alpha = x + \langle f(x) \rangle \in \dfrac{\mathbf{F}_2[x]}{\langle f(x) \rangle}$. Write every element of $\dfrac{\mathbf{F}_2[x]}{\langle f(x) \rangle}$ as a power of α.

(iii) Write $\alpha^5 + \alpha^4 + \alpha^2 + 1$ as a power of α, where α is as in (i). 5

(b) Find all the codewords of the cyclic code with generator matrix :

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Find the minimum weight of the code. How errors can the code detect and how many it correct ? 5

3. (a) Construct all possible BCH codes over $\mathbf{F}_8$ of length 8. 6

(b) Let C be any $\left[n, \dfrac{n-1}{2}\right]$ cyclic code over $\mathbf{F}_q$. Then show that C is self orthogonal if and only if C is an even like duadic code whose splitting is given by μ_{-1}. 4

4. (a) Let A_i and $A_i^{\perp}$ be the number of code-words of weight in C and $C_1^{\perp}$ respectively. Let C be a binary code generated by $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$. For $0 \leq i \leq b$, find A_i and $A_i^{\perp}$. 6

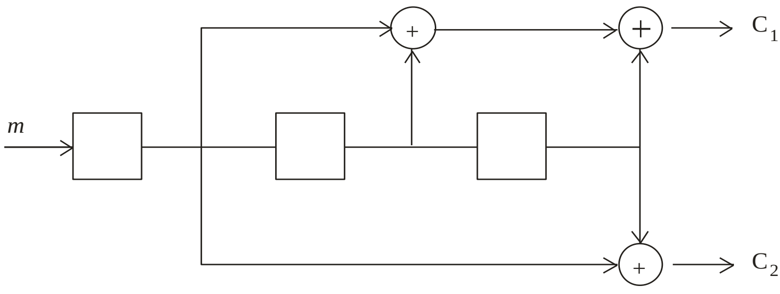
(b) Show that $\mathbf{Z}_4$ linear codes with generator matrices :

$$G_1 = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

and $$G_2 = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 1 & 2 & 0 & 1 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

are monomially equivalent. 4

5. (a) Find the convolutional code for the message 1101. The convolutional encoder is given below : 4



(b) Let C be the narrow-sense BCH code of designed distance $\delta = 5$, which has a defining set T = {1, 2, 3, 4, 6, 8, 9, 12}. Let α be a primitive 15th root of unity, where $\alpha^4 = 1 + \alpha$ and the generator polynomial C be $g(x) = 1 + x^4 + x^6 + x^7 + x^8$. If $y(x) = x + x^4 + x^7 + x^8 + x^{11} + x^{12} + x^{13}$ is received. You can use the following table : 6

0000	0	1000	α^3	1011	α^7	1110	α^{11}
0001	1	0011	α^4	0101	α^8	1111	α^{12}
0010	α	0110	α^5	1010	α^9	1101	α^{13}
0100	α^2	1100	α^6	0111	α^{10}	1001	α^{14}

6. Which of the following statements are true and which are false ? Justify your answer with a short proof or counter-example : 2×5=10

(a) Every binary Hamming code is a cyclic code.

(b) $R_{q,n} = \mathbf{F}_q[X] / \langle X^n - 1 \rangle$ is a field if and only if $n = 1$.

(c) The degree of a generator polynomial of a cyclic code is equal to the dimension of the code.

(d) There is no self-dual code of length 5.

(e) There is no (5, 3, 4) LDPC code.