No. of Printed Pages : 5 **MMTE-001**

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. Sc. (MACS)]

Term-End Examination

June, 2022

MMTE-001 : GRAPH THEORY

Time : 2 Hours *Maximum Marks : 50*

Note : *Question No.* ***1*** *is* ***compulsory****. Answer any* ***four*** *questions from Question Nos.* ***2*** *to* ***7****. Use of calculators is not allowed.*

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1. State whether the following statements are true or false. Justify your answers with a short proof or a counter-example : 10

   (i) The complement of a connected graph is connected.

   (ii) Every tree has a perfect matching.

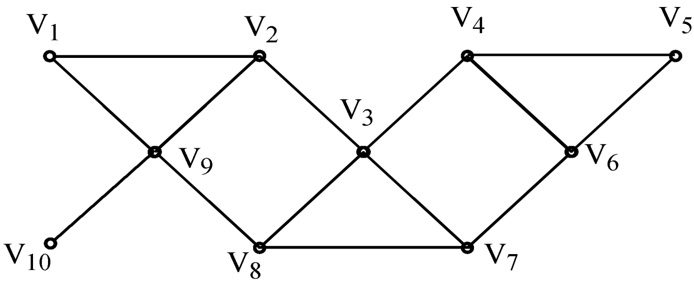
   (iii) A 2-connected planar graph is Hamiltonian.

   (iv) The Peterson graph has a 3-critical subgraph.

   (v) $\alpha\left(S_n\right) = n - 1 \quad \forall n \geq 2$

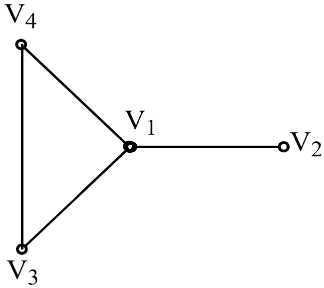
2. (a) Prove that the number of edges in an n-vertex graph with k components is at most $\binom{n-k+1}{2}$. 4

(b) Compute the diameter and radius of the following graph : 4

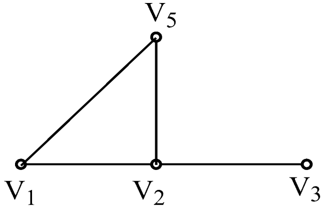


(c) There exists a self-complementary graph on 1015 vertices. True or false ? Justify. 2

3. (a) Let G and H be the graphs as shown below :



G



H

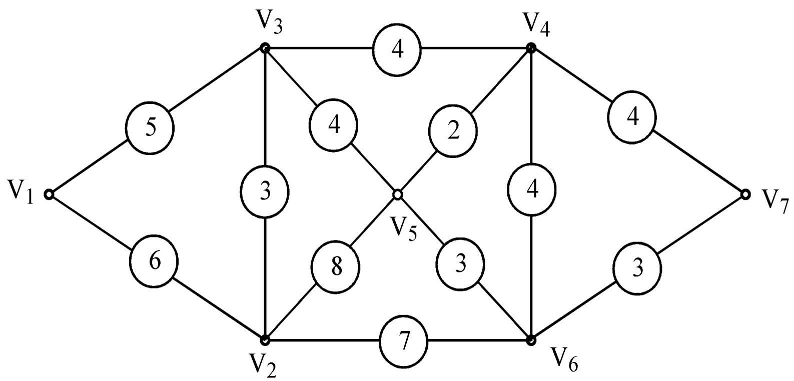
Draw GUH. Is GUH Hamiltonian ? Is it Eulerian ? Justify your answers. 4

(b) There exists a 3-edge-colourable graph on 10 vertices and 20 edges. True or false ? Justify. 3

(c) Define the terms rooted tree, binary tree and k-ary tree. 3

4. (a) Let G be a graph with the property that between every pair of vertices of G there is exactly one path. Show that G is a tree. 3

(b) Find a minimum-weight spanning tree of the following weighted graph, using Kruskal's algorithm : 5



(c) Is the complement of a Eulerian graph Eulerian ? Justify your answer. 2

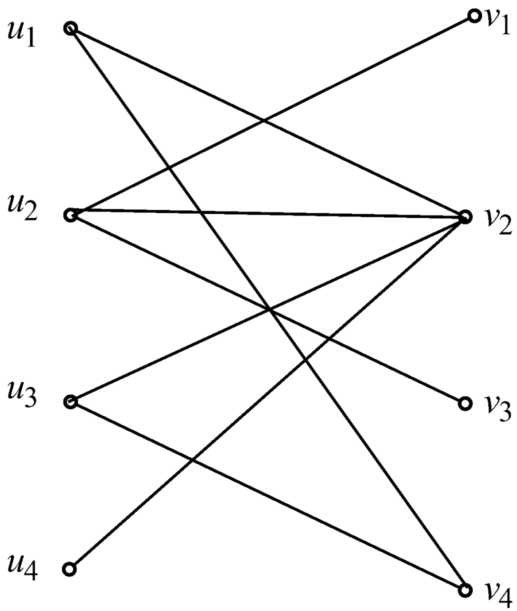
5. (a) Draw a graph G with connectivity 2, and having two vertices u and v joined by 4 internally disjoint paths. 3

(b) Check whether the Grötzsch graph is planar or not. 5

(c) Find the chromatic number of $K_{7,\,10}$. 2

6. (a) Let G be a graph with a matching M. Show that if G has no M-augmenting path, then M is maximum. 4

(b) Verify the König-Egervāry theorem for the following graph : 6



7. (a) If G is a 3-regular graph, then show that $K(G) = K'(G)$. 5

(b) Find the maximum possible value of a flow in the following network : 5