

**M.Sc. (MATHEMATICS WITH APPLICATIONS
IN COMPUTER SCIENCE)**

M.Sc. (MACS)

Term-End Examination

June, 2022

MMT-009 : MATHEMATICAL MODELLING

Time : $1\frac{1}{2}$ hours

Maximum Marks : 25

(Weightage : 70%)

Note : Attempt any **five** questions. Use of calculator is not allowed.

1. (a) State the model which is
- (i) dynamic, continuous and deterministic,
 - (ii) static, discrete and probabilistic,
 - (iii) dynamic, discrete and probabilistic,
- and give reasons in support of your answer. 3
- (b) At the end of the year 2019, Standard Restaurant decided to invest ₹ 20,000 in a portfolio of stocks and bonds; ₹ 15,000 were put into common stocks and ₹ 5,000 into corporate bonds. At the end of the year 2020, Standard Restaurant's stock and

bond holdings were worth ₹ 13,000 and ₹ 4,000 respectively. During the year 2020, ₹ 600 in cash dividends was received on stocks and ₹ 1200 as interest payments was received on bonds. What was the percentage return on

(i) Standard Restaurant's stock portfolio ?

(ii) Standard Restaurant's bond portfolio ? 2

2. (a) How would you compare the Markowitz model with Sharpe model for portfolio selection ? 2

(b) The control parameters of growth and decay of a tumour are respectively 1200 and 600 per day. Also, the damaged cells migrate due to vascularization of blood at a rate of 250 cells per day. Use logistic model to find the ratio of the growth of tumour after 30 days with the initial tumour. 3

3. The budworm population is governed by the equation

$$\frac{dN}{dt} = r_B N \left(1 - \frac{N}{K_B} \right) - p(N),$$

where r_B is the linear birth rate of the budworm and K_B is the carrying capacity. The $p(N)$ is the predation by the birds. Find out the steady states and give the stability analysis for this model. 5

4. (a) Suppose a stable of horses is constructed for 75 horses and it is decided that there will be an addition of 35 horses every year. If the mortality of horses is 5 per year, what is the population size of the horse stable after 10 years ?

2

- (b) Consider the following data :

X	3	10	4	6	2
Y	2	18	4	10	1

Use a best fit line to estimate the value of Y for X = 5.

3

5. Customers arrive at a bank in accordance with a Poisson process at the mean rate of 10 per hour and the cashier takes time to dispose a customer as exponential with a mean of 12 minutes. What is the minimum number of cashiers to be posted for ensuring a steady state distribution of work by cashiers ? For this number, calculate the

- (a) expected waiting time of a customer prior to being attended.
- (b) expected number of cashiers remaining idle.
- (c) average time a customer has to spend in the bank.

5

6. Find a linear regression equation that fits best the data given in the following table :

5

X_1	0	2	3	4	6	8
X_2	2	6	5	7	4	9
Y	2	3	2	7	6	8
