MMT-002

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

June, 2022

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Question no.* ***5*** *is* ***compulsory****. Attempt any* ***three*** *questions from questions no. 1 to 4. Use of calculators is* ***not*** *allowed.*

1. (a) Let $T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_2 \\ x_2 - x_3 \\ x_1 - x_3 \end{bmatrix}$ be a linear operator on $\mathbf{R}^3$. Find the matrix of T with respect to the ordered basis $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$.

Check whether or not T is an onto linear operator. *3*

(b) Check whether or not the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 4 \\ 1 & 4 & -2 \end{bmatrix}$ is positive definite. *2*

2. (a) Check whether or not the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$ is diagonalisable over **R**. If it is, find a unitary matrix P so that $P^{-1}AP$ is a diagonal matrix. If A is not diagonalisable, obtain its Jordan form. *3*

(b) Find the Jordan form of the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. *2*

3. (a) Find the trace of the matrix e^A, where $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. *3*

(b) Find all the least square solutions to $x - y = 1$, $x - y = 2$. *2*

4. Find the SVD of $A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$. *5*

5. Which of the following statements are *True* and which are *False* ? Give reasons for your answers. *5×2=10*

(a) If an $n \times n$ matrix is singular, then it has 0 as an eigenvalue.

(b) If all the eigenvalues of a unitary matrix are 1, then it is the identity matrix.

(c) If N is nilpotent, then e^N is also nilpotent.

(d) Every Hermitian matrix is normal.

(e) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ is the spectral decomposition of $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.