# POST GRADUATE DIPLOMA IN 

## APPLIED STATISTICS (PGDAST)

Term-End Examination

June, 2021

## MST-003 : PROBABILITY THEORY

Time : 3 Hours
Maximum Marks : 50

Note: (i) Question No. 1 is compulsory.
(ii) Attempt any four questions from the remaining (Question Nos. 2 to 7).
(iii) Use of scientific (non-programmable) calculator is allowed.
(iv) Use of formulae and statistical tables booklet for PGDAST is allowed.
(v) Symbols have their usual meanings.

1. State whether the following statements are True or False. Give reasons in support of your answers:
(a) If odds are 5 to 3 in favour of A , then the probability of occurrence of $\overline{\mathrm{A}}$ is $\frac{3}{5}$.
(b) Given that A, B and C are three mutually exclusive and exhaustive events and $\frac{1}{3} \mathrm{P}(\mathrm{C})=\frac{1}{2} \mathrm{P}(\mathrm{A})=\mathrm{P}(\mathrm{B})$, then $\mathrm{P}(\mathrm{B})=\frac{1}{6}$.
(c) A random variable X follows Binomial distribution with mean 2 and variance 6 .
(d) For a certain normal distribution the first moment about 10 is 40 . Then mean will be 50 .
(e) For the probability distribution :

| X | $\mathrm{P}(x)$ |
| :---: | :---: |
| 0 | 0.7 |
| 1 | 0.2 |
| 2 | 0.1 |

The expected value of X is 1 .
2. (a) Three light bulbs are chosen at random from 15 bulbs of which 5 are defective. Find the probability that: 4
(i) None is defective
(ii) Exactly one is defective
(iii) At least one is defective.
(b) A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box at random. Find the probability that among the balls drawn there is at least one ball of each colour. 4
(c) If A and B are independent events, then prove that $\overline{\mathrm{A}}$ and $\overline{\mathrm{B}}$ are also independent.
3. (a) A shipment of 8 similar fridges to a retailer contains 3 that are defective. If a hotel makes a random purchase of 2 of these fridges, find the probability distribution for the number of defectives. 4
(b) If the joint probability distribution of two discrete random variables is given as follows:

|  | X | 0 | 1 |
| :---: | :---: | :---: | :---: |
| $\mathbf{Y}$ |  | 2 |  |
| 1 | $3 / 28$ | $9 / 28$ | $3 / 28$ |
| 2 | $1 / 28$ | $3 / 14$ | 0 |
| 0 | 0 |  |  |

then obtain the marginal probability distributions of X and Y . Also, find $\mathrm{P}[\mathrm{X}=x \mid \mathrm{Y}=1]$ and $\mathrm{P}[\mathrm{Y}=y \mid \mathrm{X}=2]$. 6
4. (a) In five throws with a fair die, what is the probability of throwing at least 3 ones ?2
(b) There are 300 misprints randomly distributed throughout a book of 500 pages. Find the probability that a given page contains :
(i) Exactly 2 misprints
(ii) 2 or more misprints.
(c) State and prove lack of memory property for geometric distribution.
5. (a) In a normal distribution, 31\% of the items are under 45 and $8 \%$ are over 64 . Find the mode and standard deviation of the distribution.
(b) The sales tax, X, of a shopkeeper has an exponential distribution with p. d.f. :

$$
\begin{array}{rlrl}
f(x) & =\frac{1}{4} e^{-\frac{x}{4}}, & \geq 0 \\
& =0 \quad, \quad<0
\end{array}
$$

If sales tax is levied at the rate of $5 \%$, what is the probability that his sales exceed ₹ 10,000 ?
6. (a) There are three bags containing respectively 1 white, 2 red, 3 black; 2 white, 3 red, 1 black; and 3 white, 1 red, 2 black balls. A bag is chosen at random and from it two balls are drawn at random. The two drawn balls are 1 red and 1 white. What is the probability that they come from first bag?
(b) A problem in Statistics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that exactly one of them will solve the problem? 3
7. (a) The retail price of a 5 kg bag of white cement of a company varies from ₹ 200 per bag to ₹ 230 per bag. Assuming that these prices are uniformly distributed, and if price of a bag is randomly selected, what is the probability that this price is in between $₹ 210$ to ₹ 225 ? Also compute the probability that this price is less than or equal to ₹ 227. Also obtain average and standard deviation of price of bags.
(b) A confectionery company supplies jars of confectionery items to different retailers. Each jar should have 100 confectionery items. The company is aware that out of 30 jars 6 have less than 100 confectionery items. A retailer received 30 jars from the company and takes a random sample of 4 jars. What is the probability that: 5
(i) No jar has less than 100 confectionery items?
(ii) Three or more jars have less than 100 confectionery items?

