M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

June, 2021

MMTE-005 : CODING THEORY

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Answer any* ***four*** *questions from Questions No. 1 to 5. Question no.* ***6*** *is* ***compulsory****. All questions carry equal marks. Use of calculators is not allowed.*

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1. (a) Define the generator matrix of a linear code, and give an example of this. *2*

(b) Define the permutation equivalence of two linear codes, and give an example. *2*

(c) Construct a multiplication table for a finite field with 4 elements. *6*

2. (a) Let C be a non-zero cyclic code in R_n. If g(x) is the monic polynomial of minimum degree in C, prove that C is generated by g(x). *4*

(b) Construct a BCH code of length 13 and designed distance 2. Use the table below : *6*

0	0	0	0	7	1	2	2	14	0	2	0	21	1	0	1
1	0	0	1	8	2	0	2	15	2	0	0	22	0	2	2
2	0	1	0	9	0	1	1	16	0	2	1	23	2	2	0
3	0	1	2	10	1	1	0	17	2	1	0	24	2	2	1
4	1	2	0	11	1	1	2	18	1	2	1	25	2	0	1
5	2	1	2	12	1	0	2	19	2	2	2	26	0	0	1
6	1	1	1	13	0	0	2	20	2	1	1				

Table : $\mathbb{F}_{27}$ with primitive element α, where $\alpha^3 + 2\alpha + 1 = 0$

3. (a) Let C be the [5, 2] binary code generated by

$$G = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

Find the weight distribution of C. Use McWilliam's identity to find the weight distribution of $C^{\perp}$. *7*

(b) If C is a linear code, show that the minimum weight of C is the same as the minimum distance of C. *3*

4. (a) Find the convolutional code (2, 1) with generator matrix G = [D, 1+D], for the message $1 + D^2 + D^3$. *3*

(b) Show that the $\mathbb{Z}_4$ – linear codes, with generator matrices

$$G_1 = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix} \text{ and } G_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 2 \\ 0 & 2 & 0 & 2 \end{bmatrix},$$

are monomially equivalent. *5*

(c) Check whether the linear code C, with generator matrix $G = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$, is self dual. *2*

5. (a) Let C be a linear code with parity check matrix, $H = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$.

(i) List all the codewords of C.

(ii) Is C a perfect code ? Justify your answer. *8*

(b) Check whether there are duadic codes of length :

(i) 17 over $\mathbb{F}_2$

(ii) 37 over $\mathbb{F}_3$

Justify your answers. *2*

6. Which of the following statements are *True* and which are *False* ? Justify your answers. *10*

(a) Two binary codes of the same length, having the same weight distribution, are equal.

(b) If G is the generator matrix of a linear code C, and $GG^t = 0$, then C is self dual.

(c) $C = \{ (1, 0, 0), (0, 1, 0), (0, 0, 1), (0, 0, 0) \}$ is a cyclic code.

(d) $\mathbb{F}_p m \subseteq \mathbb{F}_p n$ if $m < n$.

(e) The parity check matrix of an LDPC code can be the identity matrix.