M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

June, 2021

MMTE-002 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 2 hours *Maximum Marks : 50*

Note : *Attempt any* ***four*** *questions from questions no. 1 to 5. Question no.* ***6*** *is* ***compulsory****.*

1. (a) Sort the following numbers using the QuickSort algorithm : *5*

2, 9, 7, 6, 4, 3, 8, 5

(b) Construct a Binary Search Tree for the following sequence of numbers : *5*

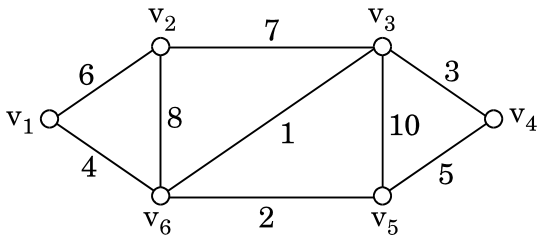
15, 50, 70, 60, 20, 90, 10, 40, 100, 35

with 40 as the value of the key for the root node.

2. (a) Find an optimal parenthesisation of a matrix chain whose sequence of dimensions is (5, 10, 3, 12, 15). *5*

(b) Let $m \in \mathbf{N} \cup \{0\}$ and $n \in \mathbf{N}$. Let the gcd of $x, y \in \mathbf{Z}$ be denoted by (x, y). Prove that $(m, n) = (m \pmod n, n)$. *5*

3. (a) Find a minimal spanning tree for the following graph using the Prim's algorithm, showing all the steps. Take v_1 as the root.



What is the weight of the minimal spanning tree you get ? Give reasons for your answer. *5*

(b) Find the Huffman code for the following set of characters :

Character	a	b	c	d	e	f
Frequency	50	30	20	12	5	3

Show all the steps of the algorithm. Also compute the number of bits required to encode the data. *5*

4. (a) Search the given pattern in the following text, using the Rabin-Karp algorithm, working modulo q = 17.

Pattern : 22

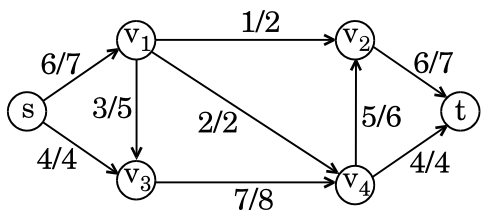
Text : 1422673673956

Also indicate all the spurious hits. *5*

(b) Illustrate, giving all the steps, the operation of the counting sort algorithm on the array A = $\langle 7, 1, 3, 1, 2, 4, 5, 7, 2, 4, 3 \rangle$. *5*

5. (a) Consider the implementation of disjoint sets using forests. Assume that the union is done by weight, i.e., the root of the tree with lesser nodes points to the root of the tree with more nodes. The following operations are applied on an initial set of elements $\{x_1, x_2, x_3, x_4, x_5\}$ union (x_1, x_2); union (x_1, x_4); union (x_2, x_3); union (x_1, x_5). Show the forest after each operation. *5*

(b) For the following network flow, draw the residual network :



Find an augmenting path p and use it to augment the flow along p. Draw the flow network of the augmented flow. *5*

6. Which of the following statements are *true* ? Give reasons for your answers, in the form of a short proof or a counter-example. *10*

(a) Dijkstra's algorithm solves the "all pairs shortest path problem".

(b) The dynamic programming design technique is used in the Radix Sort algorithm.

(c) For any $n \in \mathbf{N}$, $a \in \mathbf{Z}$, $(a, n) = 1$, we have $a^{n-1} \equiv 1 \pmod{n}$.

(d) The run time of the insertion sort algorithm depends only on the size of the data, and not on the nature of the data.

(e) Every binary heap is a binary search tree.