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MMTE-001

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. Sc. (MACS)]

Term-End Examination

June, 2021

MMTE-001 : GRAPH THEORY

Time : 2 Hours *Maximum Marks : 50*

Note : *Question No.* ***1*** *is* **compulsory**. *Answer any* ***four*** *questions from Question Nos.* ***2*** *to* ***7****. Use of calculators is not allowed.*

1. State whether the following statements are true or false. Justify your answers with a short proof or a counter-example : 10

   (i) A regular graph can have arbitrarily large diameter.

   (ii) There is a unique tree, with at least 2 vertices, whose complement is also a tree.

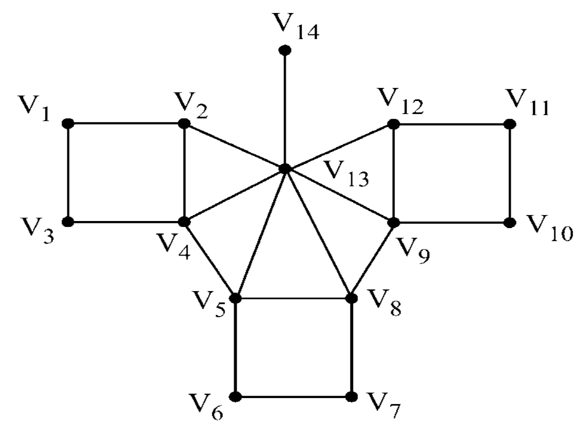
   (iii) Every graph with a cut-vertex has a cut-edge.

   (iv) Every Hamiltonian graph has a perfect matching.

   (v) If G is an n-vertex graph $(n \geq 3)$ with at most $3n-6$ edges, then G is planar.

2. (a) If u and v are the only odd vertices in a graph G, then show that G has a (u, v) path. 3

   (b) Consider the following graph G : 3



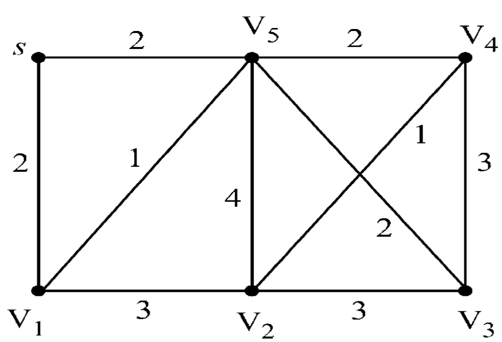
   (i) Find a *maximal* matching in G which is *not a maximum* matching.

   (ii) Does G have a perfect matching ? Justify your answer.

(c) Check whether (6, 4, 4, 4, 3, 2, 1, 1, 1) is a graphic sequence or not. If yes, find a corresponding graph. If this is not a graphic sequence, then decide whether (3, 1, 1) is a graphic sequence or not. 4

3. (a) If G is a graph with diameter d, then show that $\alpha(G) \geq \frac{d+1}{2}$. 3

(b) Use Dijkstra's algorithm to find the shortest distances from s to each of the vertices of the following weighted graph G :



Write down all the steps involved in finding the shortest paths. 5

(c) Let T be a tree with at least 3 vertices. Let T′ be the subgraph of T obtained by deleting all the leaves of T. Show that T′ is a tree. 2

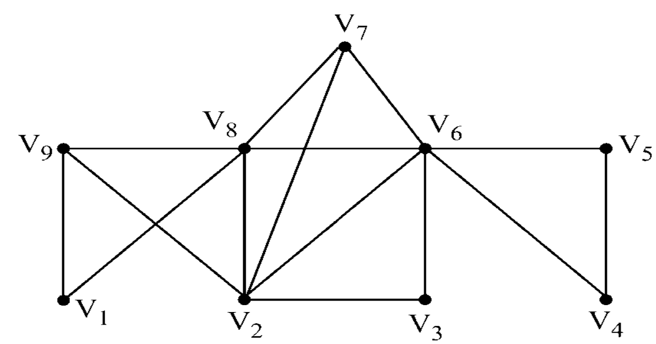
4. (a) Show that any simple graph G can be coloured with at most $\Delta(G)+1$ colours, using the Greedy Colouring Algorithm. 3

(b) Check whether the following graph is : 3

(i) Eulerian or not;

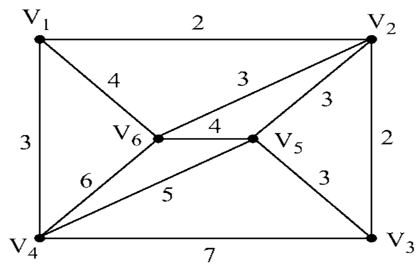
(ii) Planar or not

Justify your answers.



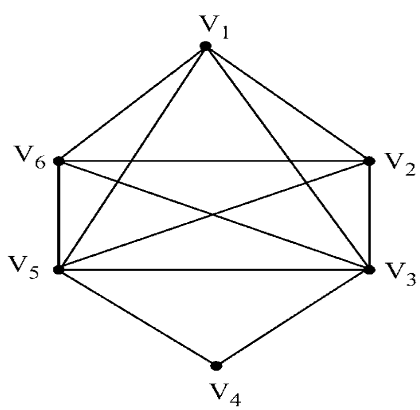
(c) Prove that a k-regular $(k > 0)$ bipartite graph has the same number of vertices in each partite set. 4

5. (a) Construct a minimum weight spanning tree for the following weighted graph, using Prim's Algorithm : 5



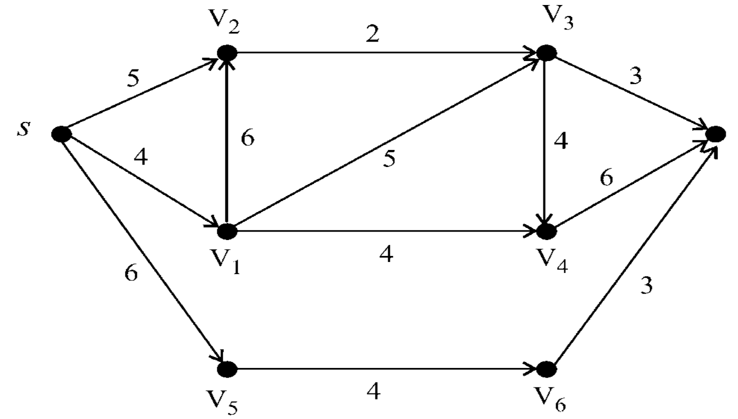
(b) What is the minimum size of a k-chromatic graph ? Justify your answer. 3

(c) Consider the following Hamiltonian graph :



Does it satisfy the Dirac's condition ? Does it satisfy Ore's condition ? Justify your answers. 2

6. (a) Find a non-zero flow on the network given below : 5



(b) Show that $K(G) \leq K'(G)$ for any graph G. 5

7. (a) Show that $K_{3,3}$ is not planar. 3

(b) Prove that a graph is connected if and only if it contains a spanning tree. 5

(c) Give an example of a non-regular 2-edge-connected graph. Justify your choice of example. 2