

**M.Sc. (MATHEMATICS WITH APPLICATIONS
IN COMPUTER SCIENCE)**

M.Sc. (MACS)

Term-End Examination

June, 2021

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ hours

Maximum Marks : 25

Note : *Question no. 1 is **compulsory**. Attempt any **three** questions from questions no. 2 to 5. Use of calculators is **not** allowed.*

1. State, giving reasons, whether the following statements are *True* or *False* : $5 \times 2 = 10$

- (a) If $f(z)$ is analytic on an open and connected set in \mathbf{C} and $|f(z)| \leq 1$ then $f(z)$ is a constant.
- (b) The radius of convergence of functions $f(z)$ and $f(iz)$ are equal.
- (c) If $\operatorname{Re} z < 0$, then $|e^{-z}| < 1$.
- (d) If $f(z) = \frac{1}{z^7} \sin z^2$, then $\operatorname{Res} f(z)$ at $z = 0$ is $\frac{-1}{6}$.

(e) Let C be the circle $|z| = 2$, then

$$\int_C \frac{e^{-z}}{z+1} dz = 2\pi i.$$

2. (a) Determine an analytic function $f(z)$ whose real part is $U(x, y) = y^2 - x^2$. 2

(b) Show that the transformation $w = \frac{1}{z}$ maps the circle $|z - 2i| = 2$ into a straight line. 3

3. (a) Evaluate the integral $\int_0^{1+i} (x^2 - iy) dz$ along the path $y = x^2$. 3

(b) Prove that $\int_{\gamma} (z - a)^n dz = 0$ for $n \neq -1$, where γ is the circle $|z - a| = r > 0$. 2

4. (a) Let $f(z) = z^2 - 6z + 9$. Find the maximum and minimum values of $|f(z)|$ in the region $|z| \leq 2$. 3

(b) Expand $f(z) = \frac{1}{(z-1)^2(z-3)}$ in a Laurent series around the point $z = 1$. Specify the region in which the expansion is valid. 2

5. Show that $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$. 5