MMT-002

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

June, 2021

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Question no.* ***5*** *is* ***compulsory****. Answer any* ***three*** *questions from questions no. 1 to 4. Use of calculators is* ***not*** *allowed.*

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1. (a) Check whether or not the matrices $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -12 & 7 \end{bmatrix}$ are similar. *2*

(b) Obtain the spectral decomposition of the matrix $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Hence obtain the square root of this matrix. *3*

2. (a) Obtain a unitary matrix whose first column is $\frac{1}{\sqrt{3}}\begin{bmatrix} i \\ -i \\ 1 \end{bmatrix}$. *2*

(b) Write all possible Jordan canonical forms for a 5×5 matrix whose only distinct eigenvalues are 1 and 2, the geometric multiplicity of 1 is two and the minimal polynomial is of degree 3. *3*

3. (a) Find the equation of the line that best fits the points (2, 0), (1, 1), (2, – 2). *2*

(b) Solve the following system of differential equations : *3*

$$\frac{dy(t)}{dt} = A\ y(t), \text{ with } y(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and}$$

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$$

4. Obtain the singular value decomposition of $A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & -1 & 4 \end{bmatrix}$. Hence obtain the Moore-Penrose of A. *5*

5. Which of the following statements are *True* and which are *not* ? Give reasons for your answers, with a short proof or a counter-example. *5*

(a) The sum of two diagonalisable linear operators on a finite-dimensional vector space is diagonalisable.

(b) Each entry of a positive definite matrix is non-negative.

(c) The rank of an $n \times n$ matrix is the number of non-zero rows in it.

(d) For $A = [a_{ij}]$, $\det A \leq a_{11}\, a_{22} \ldots a_{nn}$, where $A \in \mathbf{M}_n(\mathbf{R})$.

(e) Any two distinct eigenvectors of a matrix will be linearly independent.