## MCA (Revised)

Term-End Examination

## MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 Hours]
[Maximum Marks:100

Note: Question No. 1 is compulsory. Attempt any three from the remaining questions.

1. (a) Explain Euclid's algorithm for finding GCD of two natural numbers m and n .
(b) Write recursive binary search algorithm and analyse its complexity in worst case. 5
(c) Define Theta ( $\theta$ ) Notation. Prove that function $f(x)=5 x^{4}+7 x+3$ is $\theta\left(x^{4}\right)$. 5
(d) Design a Turing Machine (TM) for the language $L=\left\{a^{n} b^{n} c^{n}: n \geq 1\right\}$ 5
(e) Apply Karatsuba Method to multiply the following two numbers: 5026 and 3425 . 5
(f) Use mathematical induction to prove that

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\sum_{i=1}^{n} i^{2}=\frac{n(n+1)(2 n+1)}{6}
$$

(g) Write Partition Algorithm for Quick Sort. Analyse its time complexity also.
(h) If $L_{1}$ and $L_{2}$ are context free languages prove that $L_{1} \cup L_{2}$ is also context free.
2. (a) Write Dijkstra's Algorithm to find shortest path in a graph. Apply Dijkstra Algorithm for the following graph ("a" is starting vetex).

(b) Write an algorithm for topological sort. Obtain a topological ordering for the following graph:

(2)
3. (a) Differentiate between P, NP and NP complete clan of problems.
(b) Define Vertex Cover Problem (VCP). Prove that VCP is NP-Complete.
(c) Construct a Deterministic Finite Automata (DFA) for the following Language (i) defined over a set of alphabets $\sum=\{a, b\}$.
$L=\{w: W \in\{a, b\}$ and number of a in w is divisible by 3$\}$.
4. (a) Consider the following chain of matrices:
$M_{1}, M_{2}, M_{3}$ and $M_{4}$ with dimensions: $(5 \times 3)$, $(3 \times 7),(7 \times 10),(10 \times 15)$. Multiply these 4 matrices using Dynamic programming, so that it would takes minimum number of scalar multiplications. 10
(b) Write Kruskal's Algorithm to find minimum spanning free. Apply kruskal algorithm to find minimal spanning tree for the following graph:

5. (a) Explain Haltiny problem of Turing machine. 4
(b) Show that Strassen's Matrix Multiplication algorithm takes $O\left(n^{2.81}\right)$ time to multiply two matrices $A$ and $B$, each of dimensions ( $n \times n$ ).
(c) Explain the following : 10
(i) Chomsky's clarification of Grammar.
(ii) Ambiguity in Context Free Grammar (CFG).

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