No. of Printed Pages : 6

MST-003

POST GRADUATE DIPLOMA IN APPLIED STATISTICS (PGDAST)

Term-End Examination

June, 2016

01147

MST-003 : PROBABILITY THEORY

Time : 3 hours *Maximum Marks : 50*

Note :

*(i) Attempt **all** questions. Questions no. 2 to 5 have internal choices.*

(ii) Use of scientific calculator is allowed.

(iii) Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.

(iv) Symbols have their usual meaning.

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1. State whether the following statements are *True* or *False* ? Give reasons in support of your answer.

5×2=10

(a) If events A and B are independent, then $\bar{A}$ and $\bar{B}$ are also independent.

(b) Probability of a sure event is always 0·5.

(c) If the mean of a Poisson distribution is 1·2, then the central moment of order two is equal to 4·52.

(d) If variance of a random variable X is 4, then the variance of the random variable $Y = X - 3$ will be 1.

(e) If $X \sim N(\mu, \sigma^2)$, then the median of $X = 2\mu + 1$.

2. (a) Out of 52 well-shuffled playing cards, one card is drawn at random. Find the probability of getting

(i) a red card,

(ii) a face card,

(iii) a card of spades, and

(iv) a king. *4*

(b) If two dice are thrown, what is the probability that the sum is

(i) greater than 9, and

(ii) neither 10 nor 12 ? *6*

OR

(a) Both husband and wife appear in an interview for two vacancies for the same post. The probabilities of husband's and wife's selections are $\frac{2}{5}$ and $\frac{1}{5}$, respectively.

Find the probability that

(i) exactly one of them is selected,

(ii) at least one of them is selected. *5*

(b) A person speaks the truth 3 out of 4 times. A die is thrown. She reports that there is fire. What is the chance that there was fire ? 5

3. (a) The life (in hours) X of a certain type of light bulb may be supposed to be a continuous random variable with p.d.f.

$$f(x) = \begin{cases} \dfrac{A}{x^3}, & 1500 < x < 2500 \\ 0, & \text{elsewhere} \end{cases}$$

Determine the constant A and compute the probability that $1600 \le X \le 2000$. 5

(b) Two discrete random variables X and Y have $P[X = 0, Y = 0] = \frac{2}{9}$, $P[X = 0, Y = 1] = \frac{1}{9}$, $P[X = 1, Y = 0] = \frac{1}{9}$ and $P[X = 1, Y = 1] = \frac{5}{9}$.

Find the joint distribution function. Also find the marginal distribution function of X. 5

OR

(a) Let the joint density function of a two-dimensional random variable (X, Y) be

$$f(x, y) = \begin{cases} x + y, & \text{for } 0 \le x < 1 \text{ and } 0 \le y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the conditional density function of Y given X. 5

(b) The distribution of a continuous random variable X is defined by

$$f(x)=\begin{cases} x^3, & 0 < x \le 1 \\ (2-x)^3, & 1 < x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

Obtain the expected value of X. 5

4. (a) A policeman fires 6 bullets on a dacoit. The probability that the dacoit will be killed by a bullet is 0·6. What is the probability that the dacoit is still alive ? 5

(b) Assume that the chance of an individual coal miner being killed in a mine accident during a year is $\frac{1}{1400}$. Use the Poisson distribution to calculate the probability that in a mine employing 350 miners, there will be at least one fatal accident in a year. 5

OR

(a) A lot of 25 units contains 10 defective units. An engineer inspects 2 randomly selected units from the lot. He accepts the lot, if both the units are found in good condition, otherwise all the remaining units are inspected. Find the probability that the lot is accepted without further inspection. 5

(b) A proof-reader catches a misprint in a document with probability 0·8. Find the expected number of misprints in the document in which the proof-reader stops after catching the 25^{th} misprint. 5

5\. (a) For a normal distribution, the first moment about 5 is 30 and the fourth moment about 35 is 768. Find the mean and standard deviation of the distribution. 5

(b) In a normal distribution, 10% of the items are over 125 and 35% are under 60. Find the mean and standard deviation of the distribution. 5

OR

(a) Telephone calls arrive at a switchboard following an exponential distribution with parameter $\lambda = 12$ per hour. If we are at the switchboard, what is the probability that the waiting time for a call is

(i) at least 15 minutes,

(ii) not more than 10 minutes? 5

(b) Suppose that on an average 1 customer per minute arrives at a shop. What is the probability that the shopkeeper will wait more than 5 minutes before

(i) both of the first two customers arrive, and

(ii) the first customer arrives ?

Assume that waiting times follow gamma distribution. 5
