No. of Printed Pages : 4 **MMTE-001**

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00396

Term-End Examination

June, 2016

MMTE-001 : GRAPH THEORY

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Question no. 1 is* ***compulsory****. Answer any* ***four*** *questions out of the remaining six numbered 2 to 7. Electronic devices such as calculators are* ***not*** *allowed.*

1. State whether the following statements are *true* or *false*. Justify your answers with appropriate arguments or illustrations. *5×2=10*

(a) If G and H are two simple graphs and ψ is an isomorphism from G onto H, then there exist two adjacent vertices u and v in G such that ψ (u) and ψ (v) are adjacent in $\overline{H}$.

(b) Every maximal trail in an even graph is closed.

(c) A graph with n vertices and n – 1 edges is always a tree.

(d) For k $\in$ **N**, every k-regular bipartite graph has a perfect matching.

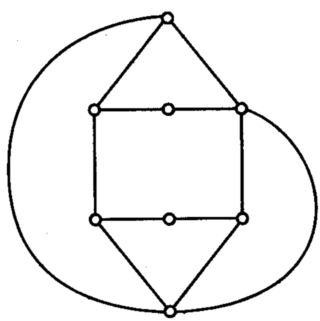
(e) K_4 is outer planar.

2. (a) Show that there is no 4-regular bipartite graph with 15 vertices. 5

(b) Prove that a graph is Eulerian if and only if it has at most one non-trivial component and all its vertices are of even degree. 5

3. (a) Let d be a list of natural numbers, of length n, and d′ be the list obtained by eliminating the largest element Δ and subtracting 1 from its next Δ largest numbers. Prove that d is graphic if and only if d′ is graphic. 6

(b) Find the chromatic number of the following graph G :

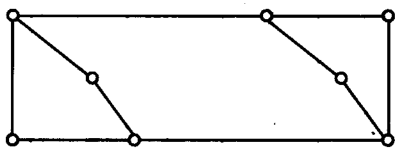


Also, give a minimal colouring of the graph. 4

4. (a) Prove that every tree with at least two vertices has at least two leaves. 3

(b) State and prove the Cayley's formula for the number of trees with n vertices. 4

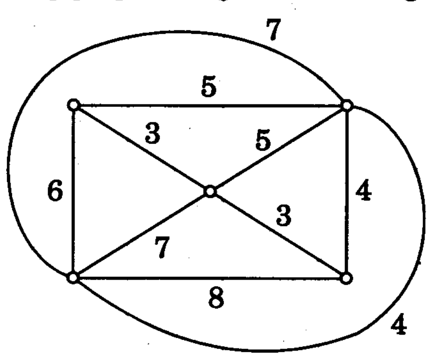
(c) Check whether the following graph is Hamiltonian :



Justify your answer. 3

5. (a) If G is a bipartite graph, then prove that the maximum size of a matching in G equals the minimum size of a vertex cover of G. 5

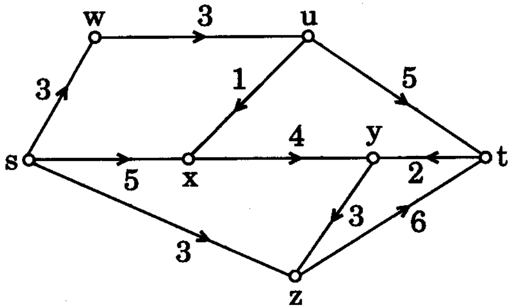
(b) Find the minimum spanning tree for the following graph using Kruskal's algorithm.



Does this graph have a unique minimal spanning tree ? Justify your answer. 5

6. (a) If G is a 2-connected graph, then show that G′, obtained by subdividing an edge of G, is also 2-connected. 4

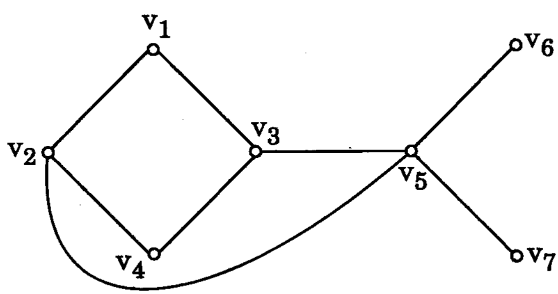
(b) Find a non-zero, feasible flow in the network given below : *6*



7. (a) Prove that $\chi(G) \leq \Delta(G) + 1$. *3*

(b) State and prove Euler's formula. *4*

(c) Identify the cut vertices and cut edges of the following graph :



Also draw the sub-graphs obtained by removing

(i) the vertex v_3,

(ii) the edge v_3v_5. *3*