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MMT-009

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

01025

June, 2016

MMT-009 : MATHEMATICAL MODELLING

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Answer any **five** questions. Use of calculator is **not** allowed.*

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1. (a) Let the returns on three securities A, B and C be 30%, 25%, and 15% respectively with $\sigma_A = 5$, $\sigma_B = 6$, $\sigma_C = 7$, $\sigma_{AB} = \sigma_{AC} = 16$ and $\sigma_{BC} = -10$. Find the standard deviation σ_P of the portfolio $P = (0{\cdot}4, 0{\cdot}1, 0{\cdot}5)$. *2*

(b) Explain each of the following with examples : *3*

(i) Reaction-diffusion model versus Advection-reaction-diffusion model

(ii) Variational matrix or Jacobian of a system of n differential equations

(iii) Optimal feasible solution of integer-linear programming problem

2. A locality is served by two malls. Each mall has two counters to serve the customers. Both the malls are equally popular and are known to have equal shares of the market. This is evident from the fact that customers arrive at each mall's serving counter at the rate of 12 customers per hour. The average time to serve a customer is 05 minutes. Customers' arrival is according to a Poisson distribution and the service time is exponential. To provide better service to the customers, the owners of the two malls decide to consolidate into a single larger mall. What is the effect of consolidation on the waiting time of customers ? *5*

3. Consider the age of candidates x and votes (in thousands) polled y in an election in a college shown in the following table :

x	23	25	28	30	21
y	7	6	9	5	2

Use a best fit line to estimate the votes for x = 24 and 26. Also estimate the error variance of the best fit. *5*

4. Consider the population model given by the difference equation

$$U_{n+1} = rU_n (1 - U_n^2), r > 0.$$

Find the steady states of the population and discuss their linear stability for

(a) $0 < r < 1$,

(b) $1 < r < 2$.

What would you expect to happen when $r > 2$? *5*

5. Do the stability analysis of the following pre-predator model formulated to study the effect of toxicants on the competing species :

$$\frac{dN_1}{dt} = r_1N_1 - \alpha_1N_1N_2 - d_1C_0N_1$$

$$\frac{dN_2}{dt} = r_2N_2 - \alpha_2N_1N_2$$

$$\frac{dC_0}{dt} = k_1P - g_1C_0 - m_1C_0$$

$$\frac{dP}{dt} = Q - hP - kPN_1 + gC_0N_1$$

under the conditions

$N_1(0) = N_{10}, N_2(0) = N_{20}, C_0(0) = 0, P(0) = P_0 > 0,$

where $N_1(t)$ = Density of prey population,

$N_2(t)$ = Density of predator population,

$C_0(t)$ = Concentration of the toxicant in the individuals of the prey population,

P = Constant environmental toxicant concentration,

r_1 and r_2 are the birth rates, α_1 and α_2 are the predation rates, Q is the exogenous rate, k, k_1 are the uptake rates, g, g_1 are the loss rates. Here r_1, r_2, α_1, α_2, k, k_1, g, g_1 and P are all positive constants. 5

6. (a) The deviation g(t) of a patient's blood glucose concentration from its optimal concentration satisfies the differential equation $\frac{d^2g}{dt^2} + 3\alpha \frac{dg}{dt} + 16\alpha^2 g = 0$ for α being a positive constant, immediately after the patient fully absorbs a large amount of glucose. The time t is measured in minutes. Identify the type (over-damped, under-damped or critically damped) of this differential equation. Find the condition on α for which the patient is normal. 3

(b) The control parameter of growth and decay of a tumour are, respectively, 1200 and 600 per day. Also the damaged cells migrate due to vascularization of blood at the rate of 200 per day. Find the ratio of the number of tumour cells after 10 days with the initial number of tumour cells. 2