No. of Printed Pages : 3

**MMT-005**

# M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
# M.Sc. (MACS)

## Term-End Examination

00316

## June, 2016

## MMT-005 : COMPLEX ANALYSIS

*Time : $1\frac{1}{2}$ hours* *Maximum Marks : 25*

---

***Note :*** *Question no. 1 is **compulsory**. Attempt any **three** questions from questions no. 2 to 5. Use of calculator is **not** allowed.*

---

**1.** State giving reasons whether the following statements are *true* or *false* : *5×2=10*

(a) The domain of definition of the function

$$f(z) = \frac{z}{z - \bar{z}}$$

is an open connected set.

(b) If $f(z) = 1 - 2z$ for $|z| \leq 1$, then $f(z)$ attains its maximum value when $z = -1$.

(c) If f(z) is bounded and continuous on the circle $C = \{z \mid |z| = 1\}$ and $\int_C f(z)\,dz = 0$, then f(z) is analytic in $\{z \mid |z| < 1\}$.

(d) If the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n z^n$ is r, then the radius of convergence of the power series $\sum_{n=1}^{\infty} n a_n z^n$ is nr.

(e) Any conformal mapping maps circles to circles and straight lines to straight lines.

**2.** (a) Show that the function

$$f(z) = \sqrt{|xy|}\ , \text{ where } z = x + iy,$$

is not differentiable at z = 0. *2*

(b) Show that

$$u(x, y) = x^3 - 3xy^2 - 3y^2 + 3x^2 + 1$$

is a harmonic function. Find its harmonic conjugate and the analytic function f(z) whose real part is u. *3*

3. (a) Show that the linear fractional transformation $w = i\frac{1-z}{1+z}$ maps the disc $|z| \leq 1$ onto the half plane $I_m w = 0$. 3

(b) Evaluate

$$\int_C \frac{(2z-3)\sin^2 z}{(z+2)(3z-1)}\,dz,$$

where $C = \{ z : |z+1| = 2 \}$. 2

4. (a) If $f(z) = u + iv$ is an entire function and $u^3 \leq 3uv^2 + 100$ on the whole complex plane, then show that f is a constant function. 3

(b) Locate the singularities of the following function and determine their type : 2

$$f(z) = \frac{2\sin z - \sin 2z}{z^4}$$

5. Evaluate

$$\int_0^\infty \frac{1}{(x^2+1)(x^2+4)}\,dx.$$ 5