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MMT-002

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

00041

June, 2016

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Question no. **5** is **compulsory**. Answer any **three** questions from questions no. 1 to 4. Use of calculators is **not** allowed.*

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1. (a) Let T be the linear operator from $\mathbf{R}^3$ to itself whose matrix with respect to the ordered basis $B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ is $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 5 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Find the matrix of T with respect to the ordered basis $B' = \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$. *3*

(b) Check that the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ is positive definite. Find the square root of A. *2*

2. (a) Write the Jordan canonical form for the matrix

$$\begin{bmatrix} 3 & 0 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}.$$ *2*

(b) Find a least square solution for the system

$3x + y = 4,\ x - y = 1,\ 2x + 2y = 1,\ 2x + y = 3.$ *3*

3. Solve the system $\frac{dy(t)}{dt} = Ay(t)$, with

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \\ 1 & -1 & 1 \end{bmatrix} \text{ and } y(0) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$ *5*

4. Write the singular value decomposition for the matrix $\begin{bmatrix} -1 & 1 \\ 0 & 1 \\ -1 & 0 \end{bmatrix}$. *5*

5. Which of the following statements are *true* and which are *false* ? Justify your answers. *5×2=10*

(a) If A is a $n \times n$ diagonalisable matrix, then A has n distinct eigenvalues.

(b) Similar matrices have the same minimal polynomial.

(c) A unitary matrix is diagonalisable.

(d) All eigenvalues of a positive definite matrix are positive.

(e) If D is a diagonalisable $n \times n$ matrix and N is a nilpotent $n \times n$ matrix, then $ND = DN$.