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BICE-027

B.Tech. – VIEP – MECHANICAL ENGINEERING / B.Tech. CIVIL ENGINEERING (BTMEVI / BTCLEVI)

Term-End Examination

00136

June, 2016

BICE-027 : MATHEMATICS-III

Time : 3 hours *Maximum Marks : 70*

Note : *Attempt any **ten** questions. All questions carry equal marks. Use of scientific calculator is permitted.*

1. Obtain the Fourier series for the function $f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases}$ and hence show that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}.$$ 7

2. Find the half-range cosine series of the function $f(x) = x \sin x,\ 0 < x < \pi$. 7

3. Obtain the Fourier series for the function

$$f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2 - x), & 1 \le x \le 2 \end{cases}.$$ 7

4. Obtain the Fourier series to represent

$f(x) = \frac{1}{4}(\pi - x)^2$ in the interval $0 \le x \le 2\pi$ and

hence show that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots = \frac{\pi^2}{12}$. 5+2=7

5. Solve the partial differential equation

$$x^2p + y^2q = (x + y)\, z.$$ 7

6. Solve : 7

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 2\,(y - x) + \sin\,(x - y).$$

7. Find the Fourier sine transform of $\frac{e^{-9x}}{x}$. Hence find the Fourier sine transform of $\frac{1}{x}$. 4+3=7

8. Find the Fourier transform of the function

$$F(x) = \begin{cases} x, & \text{for} \quad |x| < a \\ 0, & \text{for} \quad |x| > a \end{cases}$$ 7

9. Solve the P.D.E. by separation of variables method : 7

$$u_{xx} = u_y + 2u,\ u\,(0, y) = 0,\ \frac{\partial}{\partial x}u\,(0, y) = 1 + e^{-3y}.$$

10. Use the method of separation of variables to solve the equation $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$. 7

11. A tightly stretched string with fixed end points x = 0 and x = l is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points an initial velocity $\lambda x (l - x)$, find the displacement of the string at any distance x from one end at any time t. 7

12. Find the temperature in a bar of length 2 whose ends are kept at zero and lateral surface insulated, if the initial temperature is $\sin \frac{\pi x}{2} + 3 \sin \frac{5\pi x}{2}$. 7

13. Solve the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$,

subject to the boundary conditions

$u (0, y) = u (l, y) = u (x, 0) = 0$ and

$u (x, a) = \sin \frac{n\pi x}{l}$. 7

14. An infinitely long plane uniform plate is bounded by two parallel edges and an end at right angles to them. The breadth is π. This end is maintained at temperature u_0 at all points and other edges are at zero temperature. Determine the temperature at any point of the plate in the steady state. 7

15. Find the current i and voltage v in a transmission line of length l, t seconds after the ends are suddenly grounded given that $i(x, 0) = i_0$ and $v(x, 0) = v_0 \sin\left(\frac{\pi x}{l}\right)$ and that R and G are negligible. 7