

**B.Tech. Civil (Construction Management) /
B.Tech. Civil (Water Resources Engineering) /
B.Tech. (Aerospace Engineering)**

Term-End Examination

June, 2016

00590

ET-102 : MATHEMATICS – III

Time : 3 hours

Maximum Marks : 70

Note : Attempt any ten questions. All questions carry equal marks. Use of scientific calculator is allowed.

1. Show that the sequence $\langle (-1)^n \rangle$, for all $n \in \mathbb{N}$, is not a Cauchy sequence, but $\langle [(-1)^n]^2 \rangle$ is convergent. 7

2. Test for convergence the series $\sum S_n$, where $S_n = \sqrt{\frac{n-1}{\sqrt{n^2+1}}} x^n$, for all values of x . 7

3. Find the half-range cosine series for the function $f(x) = \begin{cases} x & \text{for } 0 < x < \frac{\pi}{2} \\ (\pi - x) & \text{for } \frac{\pi}{2} < x < \pi. \end{cases}$ 7

4. Show that the series $\sum \frac{1}{n} \left(\frac{2+n}{e} \right)^n$ is divergent. 7

5. (a) Find Laplace Transform of $\left(\frac{\sin 2t}{t}\right)$.

(b) If L^{-1} represents Inverse Laplace Transform, show that

$$L^{-1} \left[\frac{1}{(s+1)(s^2+1)} \right] = \frac{1}{2} [\sin t - \cos t + e^{-t}]. \quad 3+4$$

6. Solve, using Laplace Transforms,

$$\frac{d^3 y}{dt^3} + 3 \frac{d^2 y}{dt^2} + 3 \frac{dy}{dt} + y = t^2 e^{-t} \text{ given that}$$

$$y(0) = 1, y^{(1)}(0) = 0, y^{(2)}(0) = -2. \quad 7$$

7. A particle falls from rest in a medium in which the resistance is λv^2 per unit mass (v being velocity of the particle at time t). Prove that the distance fallen in time t is $\frac{1}{\lambda} \log [\cosh (t\sqrt{g\lambda})]$. 7

8. (a) Solve the differential equation

$$2(1 - xy) \frac{dy}{dx} = y^2.$$

(b) Find the particular integral of the differential equation

$$(D^3 - 2D^2 - 19D + 20)y = xe^x. \quad 4+3$$

9. Solve that $x = 0$ is a regular singular point of the equation

$$(x + x^2 + x^3) y'' + 3x^2 y' - 2y = 0.$$

Find the indicial equation, its roots and the recurrence relation. 7

10. Solve $(xy^2 - 2x^4)p + (2y^4 - x^3y)q - 9z(x^3 - y^3) = 0$,
using Lagrange's method. 7

11. Find the solution of the heat conduction problem

$$\frac{\partial^2 u}{\partial x^2} - 9 \frac{\partial u}{\partial t} = 0, \quad 0 < x < 3, \quad t > 0$$

with

$$u(0, t) = 0 = u(3, t)$$

$$u(x, 0) = 2 \sin \frac{\pi x}{3} - \sin \pi x + 4 \sin 3\pi x,$$

using separation of variable method. 7

12. (a) Find the magnificent coefficient of Conformal Transformation $w = z^3$ at the point $(2 - i)$.

- (b) Find the bilinear transformation that maps $i, 1, -1$ into $1, 0, \infty$. 3+4

13. Show that the function

$$v = -9x^2y + 2y + 3y^3$$

is harmonic and find its conjugate function. 7

14. Find

- (a) the Laurent's expansion of the function

$$f(z) = \frac{2z^3 + 1}{z + z^2}$$

in the annulus $0 < |z| < 1$.

- (b) Taylor's series expansion of $f(z)$ valid in the neighbourhood of the point $z = 1$. 3+4

15. Find the value of

$$\int_C \frac{10z - 3}{(z + 1)^2 (3z + 4)},$$

where C is the circle $|z + i| = \sqrt{3}$.

7

16. Evaluate :

$$\int_0^\pi \frac{1 + \cos \theta}{3 + 2\cos \theta} d\theta,$$

using the method of complex variables.

7

17. Apply the Hurwitz–Routh criterion to determine the stability of the system whose characteristic equation is given by

7

$$s^5 - 2s^4 + 3s^3 + 5s^2 - 11s - 13 = 0.$$
