

**B.Tech. Civil (Construction Management) /  
B.Tech. Civil (Water Resources Engineering) /  
B.Tech. (Aerospace Engineering) /**

**BTCLEVI / BTMEVI / BTELVI / BTECVI / BTCSVI**

**Term-End Examination**

00090

**June, 2016**

**ET-101 (B) : MATHEMATICS – II  
(PROBABILITY AND STATISTICS)**

*Time : 3 hours*

*Maximum Marks : 70*

---

*Note : All questions are compulsory. Attempt any two parts out of the three in each question. Each question carries equal marks. Use of scientific calculator is permitted.*

---

1. (a) For the two events A and B show that  
 $P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$ .
- (b) Suppose an assembly plant receives its voltage regulators from three different sources, 60% from  $B_1$ , 30% from  $B_2$  and 10% from  $B_3$ . Let 95%, 80%, and 65% of the supply received respectively from the sources  $B_1$ ,  $B_2$  and  $B_3$  perform as per specifications laid. If A is the event that a voltage regulator received at the plant performs as per specification, then find  $P(A)$ .

- (c) The joint density function of  $X$  and  $Y$  is given by  $f(x, y) = 4xy e^{-(x^2+y^2)}$ ;  $x, y \geq 0$ .

Test whether  $X$  and  $Y$  are independent. Also find the conditional density function of  $X$  given  $Y = y$ .  $2 \times 7 = 14$

2. (a) Seven coins are tossed and the number of heads are noted. The experiment is repeated 128 times and the following distribution is obtained.

No. of Heads :	0	1	2	3	4	5	6	7
Frequency :	7	6	19	35	30	23	7	1

Fit a binomial distribution assuming the coin to be unbiased and test the goodness of fit.

- (b) Define a Poisson variate. Find its mean and variance. Show that the sum of two independent Poisson variates is a Poisson variate.
- (c) A multiple-choice quiz has 200 questions each with 4 possible answers of which only 1 is the correct answer. What is the probability that sheer guess-work yields 25 to 30 correct answers for 80 of the 200 problems about which the student has no knowledge ?  $2 \times 7 = 14$

3. (a) Find the correlation coefficient  $\rho$  between the variables (X, Y) defined by the p.d.f.

$$f(x, y) = \frac{1}{8}(x + y), 0 \leq x \leq 2, 0 \leq y \leq 2.$$

- (b) If F has F-distribution with  $(v_1, v_2)$  d.f., then show that  $1/F$  follows an F distribution with  $(v_2, v_1)$  d.f.

- (c) Sixteen numbers are selected independently and at random from the interval  $[0, 1]$ . Find the probability that the mean of these sixteen numbers does not exceed 0.6.  $2 \times 7 = 14$

4. (a) State and prove central limit theorem.

- (b) The number of items cleared by an assembly line during a week is a random variable with mean 50 and variance 25.

(i) What is the probability that items cleared this week will exceed 75 ?

(ii) What can be said about the probability that this week's clearance will be between 40 and 60 ?

- (c) Is  $\bar{X}^2$  an unbiased estimator for  $\mu^2$ ,  $\mu$  being population mean ? If not find its bias.  $2 \times 7 = 14$

5. (a) A random sample of size  $n$  is taken from a geometric distribution with parameter  $p$ . Find the moment estimator and MLE for  $p$ . Are they same ?

- (b) The test runs with six models of an experimental engine showed that they operated respectively for 24, 28, 21, 23, 32 and 22 minutes with a gallon of fuel. Obtain a 99% C.I. for the average run time of engine with a gallon of fuel.
- (c) The following table gives the sample data on the number of defective castings produced by five different moulds.

Moulds :	I	II	III	IV	V
Defective Castings :	14	33	21	17	25
Sample size :	100	200	180	120	150

On the basis of the data can we say that the proportion of defectives is same for different moulds ? Use  $\alpha = 0.05$ .

$2 \times 7 = 14$