BACHELOR OF COMPUTER APPLICATIONS (BCA) (Revised)
Term-End Examination
June, 2016

04336

BCS-012 : BASIC MATHEMATICS

Time : 3 hours *Maximum Marks : 100*

*Note : Question number 1 is **compulsory**. Attempt any **three** questions from the remaining questions.*

1. Attempt *all* parts :

   (a) Show that

   $$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a).$$ 5

   (b) If $A = \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$, $B = \begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix}$ and

   $(A+B)^2 = A^2 + B^2$, find a and b.   5

(c) Use the principle of mathematical induction to show that $2 + 2^2 + ... + 2^n = 2^{n+1} - 2$ for each natural number n. 5

(d) Find the 10th term of the harmonic progression $\dfrac{1}{7}, \dfrac{1}{15}, \dfrac{1}{23}, \dfrac{1}{31}, ...$ 5

(e) If Z is a complex number such that $|Z - 2i| = |Z + 2i|$, show that $Im(Z) = 0$. 5

(f) Find the quadratic equation whose roots are $2 - \sqrt{3}, 2 + \sqrt{3}$. 5

(g) If $y = ln\left[e^x\left(\dfrac{x-2}{x+2}\right)^{3/4}\right]$, find $\dfrac{dy}{dx}$. 5

(h) Evaluate : 5

$$\int \frac{dx}{\sqrt{x} + x}$$

2. (a) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, show that

$A^2 - 4A - 5I_3 = 0$. Hence obtain A^{-1} and A^3. 10

(b)   If $A = \begin{bmatrix} 3 & 4 & -5 \\ 1 & 1 & 0 \\ 1 & 1 & 5 \end{bmatrix}$, show that A is row equivalent to I_3.   *5*

(c)   Use Cramer's rule to solve the following system of equations :   *5*

$$x + 2y + 2z = 3$$
$$3x - 2y + z = 4$$
$$x + y + z = 2$$

3. (a)   Find the sum of an infinite G.P. whose first term is 28 and fourth term is $\dfrac{4}{49}$.   *5*

(b)   If $x = a + b$, $y = a\omega + b\omega^2$, $z = a\omega^2 + b\omega$ (where ω is a cube root of unity and $\omega \neq 1$), show that $xyz = a^3 + b^3$.   *5*

(c)   If the roots of $ax^3 + bx^2 + cx + d = 0$ are in A.P., show that

$$2b^3 - 9abc + 27a^2d = 0.$$ *5*

(d)   Solve the inequality

$$\frac{5}{|x - 3|} < 7.$$ *5*

4. **(a)** Determine the values of x for which
$f(x) = 5x^{3/2} - 3x^{5/2}$, $x > 0$ is

    (i)   increasing

    (ii) decreasing.   *5*

(b) Find the points of local extrema of

$$f(x) = \frac{3}{4}x^4 - 8x^3 + \frac{45}{2}x^2 + 2015.$$ *5*

(c) Evaluate :   *5*

$$\int \frac{x^2}{(x+2)^3}\, dx$$

(d) Find the area bounded by the curves $y = x^2$
and $y^2 = x$.   *5*

5. **(a)** For any vectors show that

$$|\ \vec{a} + \vec{b}\ | \le |\ \vec{a}\ | + |\ \vec{b}\ |.$$ *5*

(b) Find the shortest distance between

$$\vec{r} = (1 + \lambda)\ \hat{i} + (2 - \lambda)\ \hat{j} + (1 + \lambda)\ \hat{k} \text{ and}$$

$$\vec{r} = 2(1 + \mu)\ \hat{i} + (1 - \mu)\ \hat{j} + (-1 + 2\mu)\ \hat{k}.$$ *5*

(c) A man wishes to invest at most ₹ 12,000 in Bond A and Bond B. He must invest at least ₹ 2,000 in Bond A and at least ₹ 4,000 in Bond B. If Bond A gives return of 8% and Bond B that of 10%, find how much money be invested in the two bonds to maximize the return. *10*

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