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MMT-009

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

00586

June, 2014

MMT-009 : MATHEMATICAL MODELLING

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Answer any **five** questions. Use of calculator is **not** allowed.*

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1. (a) What are the factors to classify the mathematical models in population dynamics ? Write any four of them. On the basis of growth rate, classify population models giving an example of each. *2*

(b) Find the least square line that best fits the following data : *3*

X	20	22	24	26	28	30	32
Y	50	55	40	35	30	60	25

2. Do the stability analysis of the following model formulated to study the effect of toxicant on prey-predator population : 5

$$\frac{dN_1}{dt} = r_0N_1 - r_1C_0N_1 - bN_1N_2$$

$$\frac{dN_2}{dt} = -d_0N_2 - d_1V_0N_2 + \beta_0bN_1N_2$$

$$\frac{dC_0}{dt} = k_1P - g_1C_0 - m_1C_0$$

$$\frac{dV_0}{dt} = k_2P - g_2V_0 - m_2V_0$$

$$\frac{dP}{dt} = Q - hP - kP(N_1 + N_2) + gC_0N_1 + lV_0N_2 .$$

Here r_0, r_1, b, d_0, d_1, β_0, k_1, k_2, g_1, g_2, m_1, m_2, Q, h, k, g, l are all positive constants.

$N_1(t)$ = Density of prey population.

$N_2(t)$ = Density of predator population.

$C_0(t)$ = Concentration of the toxicant in the individuals of the prey population.

$V_0(t)$ = Concentration of the toxicant in the individuals of the predator population.

$P(t)$ = Concentration of the toxicant in the environment.

Q = Constant input rate, h = decay rate.

$k =$ Ingestion rate of toxicant by the populations.

$g, l =$ return rate of toxicant in the environment after the death of the populations, assuming that toxicant is non-degradable.

r_0, r_1 are the birth rates, d_0 is death rate, b is predation rate, β_0 is conversion coefficient, m_1, m_2 are depuration rates, k_1, k_2 are uptake rates, g_1, g_2 are loss rates.

Also interpret the solutions obtained.

3. Customers arrive at the supermarket store counter in accordance with a Poisson process at the mean rate of 15 per hour and the service time by the salesperson is exponential with a mean of 12 minutes. What is the minimum number of counters to be set up for ensuring a steady-state distribution ? For this number, calculate (1) the expected waiting time of a customer in queue prior to being attended, (2) the expected number of customers waiting in the store, (3) average time a customer has to spend in the store, (4) probability that a customer is attended without having to wait in the store. *5*

4. Consider the budworm population dynamics governed by the equation $\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - x$ where k, the carrying capacity, and r the birth rate, of the budworm population x(t) are positive parameters. Find the steady states and use the perturbation to do the stability analysis of the equation for $0 < r < 1$. 5

5. (a) The deviation g(t) of a patient's blood glucose concentration from its optimal concentration satisfies the differential equation $4\frac{d^2g}{dt^2} + 8\alpha\frac{dg}{dt} + (4\alpha)^2 g = 0$ for α, a positive constant, immediately after the patient fully absorbs a large amount of glucose. The time t is measured in minutes. Identify the type of this differential equation (overdamped, underdamped, critically damped) and find the deviation g(t) at any time t. Also find the condition on α for which the patient is normal. 3

(b) Given below are different problems related to tumour growth. Identify the type of model (deterministic, continuous, stochastic or discrete) which is most appropriate in each of the following situations. Give reasons for your answer. 2

(i) Effect of the treatment given at regular intervals.

(ii) Effect of drugs on a patient who is given the drug for a given duration of time.

(iii) Effect of radiation on tumour cells; some cells may continue to grow but some may be damaged.

(iv) Finding the time taken for a tumour to double in size.

6. (a) Compare the risk of two securities 1 and 2 whose return distributions are given below : 3

Possible rates of returns for Security		Associated Probability
1	2	$P_{1j} = P_{2j}$
0·11	0·18	0·42
0·17	0·16	0·15
0·10	0·11	0·30
0·19	0·09	0·13

(b) Give two limitations of the regression model. 2