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MMT-007

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

00926

M.Sc. (MACS)

Term-End Examination

June, 2014

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 hours *Maximum Marks : 50*

(Weightage 50%)

Note : *Question 1 is **compulsory**. Do any **four** questions out of question nos. 2 to 7. All computations may be kept to **3** decimals. Use of calculators is **not** allowed.*

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1. State whether the following statements are *true* or *false*. Justify your answer with the help of a short proof or a counter example. *2×5=10*

(a) For the differential equation $(1 - x^2)y'' - 2xy' + p(p + 1)y = 0$, $x = 1$ is a regular singular point.

(b) If $H_n(x)$ is the n^{th} Hermite polynomial,

then $\int_{-\infty}^{\infty} e^{-x^2} P(x) H_n(x)\, dx = 2^n . n! \sqrt{\pi}$, for

any polynomial P(x) of degree $k < n$.

(c) The inverse Fourier transform

$$\mathscr{F}^{-1}\left(\frac{1}{\alpha^2 + i\alpha + 2}\right) = \frac{1}{3}[H(x)e^{-x} + H(-x)e^{2x}].$$

(d) The interval of absolute stability of the Taylor series method

$$y_{i+1} = y_i + hy_i' + \frac{h^2}{2} y_i''$$

for the initial value problem

$y' = \lambda y,\ y(x_0) = y_0$ is $]-2, 0[$.

(e) The order of the method

$$u_{xx} = \frac{1}{h^2}[u(x+h, y) - 2u(x, y) + u(x-h, y)]$$

is three.

2. (a) Solve by the method of Laplace transform :

$y'' + 2y' - 3y = 3,\ y(0) = 4,\ y'(0) = -7$ *4*

(b) Find the power series solution near $x = 0$ of the differential equation

$$9x(1-x)y'' - 12y' + 4y = 0.$$ *6*

3. (a) Find the solution of the boundary value problem

$$\nabla^2 u = x^2 + y^2, \ 0 \le x \le 1, \ 0 \le y \le 1$$

subject to the boundary conditions

$u = \frac{1}{12}(x^4 + y^4)$ on the lines

$x = 1, y = 0, y = 1$

and $12u + \frac{\partial u}{\partial x} = x^4 + y^4 + \frac{x^3}{3}$ on $x = 0$

using the five point formula. Assume $h = \frac{1}{2}$ along both axes. Use central difference approximation in the boundary conditions. *6*

(b) Derive the method

$$y_{i+1} = a_1 y_i + a_2 y'_{i-1} + h b_0 y'_{i+1}$$

for solving the initial value problem $y' = f(x, y), y(x_0) = y_0$. Find the truncation error and the order of the method. *4*

4. (a) Obtain the approximate value of y(0·8) for the initial value problem

$$y' = x^2 + y^2, \; y(0) = 1$$

using the predictor-corrector method

$$P : y_{i+1} = y_{i-1} + \frac{4h}{3}(2f_i - f_{i-1} + 2f_{i-2})$$

$$C : y_{i+1} = y_{i-1} + \frac{h}{3}(f_{i+1} + 4f_i + f_{i-1})$$

with h = 0·2. Calculate the starting values using the Euler method with the same step length. Perform two corrector iterations per step. *6*

(b) Find Laplace inverse of $\frac{1}{\sqrt{2s+3}}$. *4*

5. (a) Find the solution of the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

subject to the conditions

u(x, 0) = 0, u(0, t) = 0 and u(1, t) = t,

using implicit Crank – Nicolson method with $h = \frac{1}{2}$ and $k = \frac{1}{8}$. Integrate for two time levels. *5*

(b) Construct Green's function for the boundary value problem

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 0, \quad 0 < x < \pi/2$$

$$y(0) = 0, \quad y(\pi/2) = 0.$$ 5

6\. (a) If $f'(x_k)$ is approximated by

$$f'(x_k) = af(x_{k+1}) + bf(x_{k+1}).$$

find the values of a and b. What is the order of approximation ? 3

(b) Using Laplace transform, solve the integro-differential equation

$$y' + 4y + 4\int_0^1 y(\tau)\, d\tau = 5, \; y(0) = 4.$$ 4

(c) Express $J_2(x)$ and $J_3(x)$ in terms of $J_0(x)$ and $J_1(x)$. 3

7\. (a) Solve the boundary value problem

$$y'' = xy$$

$$y(0) + y'(0) = 1, \; y(1) = 1.$$

Take $h = \frac{1}{3}$ and use second order method. 5

(b) Show that

$$P'_{n+1}(x) = (2n + 1)\, P_n(x) + P'_{n-1}(x)$$

where $P_n(x)$ is a Legendre polynomial of degree n. *3*

(c) Using the substitution $z = \sqrt{x}$, reduce the given equation to Bessel equation and hence find its solution.

$$xy'' + y' + \frac{y}{4} = 0.$$ *2*