No. of Printed Pages : 3

**MMT-002**

# M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

## M.Sc. (MACS)

00896

## Term-End Examination

## June, 2014

## MMT-002 : LINEAR ALGEBRA

*Time : $1\frac{1}{2}$ hours* *Maximum Marks : 25*

*(Weightage : 70%)*

***Note :*** *Question no. 5 is **compulsory**. Answer any **three** questions from 1 to 4. Calculators are **not** allowed.*

1. (a) Let $T : \mathbf{R}^3 \to \mathbf{R}^2 : T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x + 2y + z \\ x - y - 2z \end{bmatrix}$.

Find the matrix of T with respect to the bases $\left\{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right\}$ and $\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\}$ of $\mathbf{R}^3$ and $\mathbf{R}^2$, respectively. *2*

(b) Prove that given an $m \times n$ matrix M, then $\alpha \in \mathbf{R}^n$ is a least squares solution of $Mx = y$ if and only if $M^* M\alpha = M^*y$. *3*

2. (a) Write all possible Jordan canonical forms for a $4 \times 4$ matrix whose minimal polynomial is $(x-1)(x-2)$. $2\frac{1}{2}$

(b) Write the spectral decomposition of

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}.$$ $2\frac{1}{2}$

3. (a) Check whether or not $\begin{bmatrix} 2 & -3 & 3 \\ 5 & 5 & 2 \\ 0 & 3 & 0 \end{bmatrix}$ is unitarily diagonalisable. 2

(b) Consider the predator-prey matrix of two populations given by $\begin{pmatrix} 0{\cdot}38 & 0{\cdot}24 \\ -0{\cdot}36 & 1{\cdot}22 \end{pmatrix}$. Check whether the populations perish with time or not. 3

4. Construct the SVD of $\begin{bmatrix} 0 & -3 & -1 \\ 0 & 1 & -3 \end{bmatrix}$. 5

5. Which of the following statements are true, and which are not ? Give reasons for your answers. *10*

(i) Eigenvectors corresponding to the same eigenvalues of a matrix are always linearly dependent.

(ii) If all the eigenvalues of a matrix A are zero, then A is similar to the zero matrix.

(iii) The sum of two normal matrices of order n is normal.

(iv) If A is a matrix with determinant 1, then A is a unitary matrix.

(v) If the characteristic polynomial of a matrix is $(x-3)^2(x-2)^2$, then its minimal polynomial can be $(x-3)^2$.