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ET-101(B)

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering) / BTCLEVI / BTMEVI / BTELVI / BTECVI / BTCSVI

01475

Term-End Examination
June, 2014

ET-101 (B) : MATHEMATICS – II
(PROBABILITY & STATISTICS)

Time : 3 hours *Maximum Marks : 70*

Note : *Attempt any* ***seven*** *questions. All questions carry equal marks. Use of calculator is permitted.*

1. (a) Can two events be (i) mutually exclusive and exhaustive, (ii) exhaustive and independent ? Justify your answer by giving an example in each case. *5*

   (b) A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card. *5*

2. (a) State and prove the theorem of total probability. 5

(b) In a bolt factory, machines B_1, B_2 and B_3 manufacture 25, 35 and 40 percent of the total output respectively. Of their outputs 5, 4 and 2 percent, respectively are defective bolts. A bolt is chosen at random and found to be defective. What is the probability that the bolt came from machine B_3 ? 5

3. (a) Suppose that a radio tube inserted into a certain type of set has a probability of 0·2 of functioning more than 500 hours. If we test 20 such tubes, what is the probability that at least 3 of these tubes function more than 500 hours ? 5

(b) Define a Poisson variate. Derive its distribution as a limiting case of binomial variate. 5

4. (a) Define a continuous random variable. Give example. If the pdf of X is $f(x) = \frac{1}{2} \sin x$, $x \in (0, \pi)$, $f(x) = 0$, otherwise, find its S.D. σ and the mean absolute deviation δ from the mean. 5

(b) Define a normal variate. Show that its distribution is symmetric. 5

5. (a) Define product-moment correlation coefficient between two random variables (X, Y). Prove that it is zero when the variables are independent, but the converse is not true. 5

(b) The joint pdf of X and Y is given by

$$f(x, y) = \begin{cases} 2e^{-(x + 2y)}, & 0 < x < \infty, 0 < y < \infty \\ 0 \quad , & \text{otherwise} \end{cases}$$

Find : (a) P {X > 1, Y < 1} and (b) P {X < Y}. 5

6. (a) If X_1 and X_2 are two independent random variables, each distributed uniformly in the interval [0, a], where a > 0 is a constant, find the joint distribution of $X_1 + X_2$ and $X_1 - X_2$. 5

(b) Under what conditions can Poisson probabilities be approximated by the normal probability ? Use normal approximations to compute the probability Pr $(6 \leq X \leq 10)$ where X follows Poisson distribution with parameter $\lambda = 9$. 5

7. (a) Define unbiased estimator. Let s^2 be the sample variance based on random sample $(X_1, X_2, ..., X_n)$ from the population. Then show that $E(s^2) = \sigma^2$, the population variance. 5

(b) Let $X_1, X_2, ..., X_n$ be a random sample from a population having mean μ and variance σ^2. Show that $\overline{X} = \frac{2}{n^2} \sum_{i=1}^{n} i X_i$ is the consistent estimator of μ. 5

8. (a) The following are 24 determinations of daily emission of sulphur oxides (in tons) of an industrial plant :

52	43	41	71	47	55
62	56	33	61	54	50
45	63	53	65	66	70
52	38	46	44	60	56

Obtain a 95% confidence interval of the actual average daily emission of oxides by the plant using the above data. 5

(b) The test runs with six models of an experimental engine showed that they operated respectively for 24, 28, 21, 23, 32 and 22 minutes with a gallon of fuel. Obtain a 99% confidence interval for the average run time of engine with a gallon of fuel. 5

9. (a) Let p be the probability of rolling a six with a given dice. If the dice is fair then P = 1/6. If 30 sixes were observed in 150 independent castings with this dice, can we say that the dice is fair ? Use $\alpha = 0{\cdot}01$. *5*

(b) The following is the sample data that gives the measurements on the heat producing capacity (in millions of calories/ton) of coal from two mines :

Mine I	Mine II
8260	7950
8130	7890
8350	7900
8070	8140
8340	7920
	7840

Assume that the variance of heat producing capacities of coal in two mines is nearly the same. Test at 5% level whether or not the two mines supply coal with the same heat capacities. *5*