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ET-101(A)

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering) / BTCLEVI / BTMEVI / BTELVI / BTECVI / BTCSVI

Term-End Examination

01355

June, 2014

ET-101 (A) : MATHEMATICS – I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are* ***compulsory****. Use of calculator is allowed.*

1. Answer any ***five*** of the following : *5×4=20*

(a) Evaluate any ***one*** of the following limits :

(i) $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$

(ii) $\lim_{x \to 0} \frac{\sqrt{1 - x^2 + x^2} - \sqrt{1 + x^2}}{4^x - 1}$

(b) If $y = e^{\tan^{-1} x}$, then prove that

$(1 + x^2) y_{n+2} + [2(n + 1) x - 1] y_{n+1} + n(n + 1) y_n = 0$

(c) If $2^x + 2^y = 2^{x+y}$, then find the value of $\frac{dy}{dx}$ at $x = y = 1$.

(d) If $x = \cos(ln\ y)$, then show that $(1 - x^2)\, y_2 - xy_1 = y$.

(e) If $x = r\cos\theta$, $y = r\sin\theta$, and $z = z$, find $\frac{\partial(x, y, z)}{\partial(r, \theta, z)}$.

(f) A particle of unit mass moves in a straight line in a resisting medium which produces a resistance Kv. If the particle starts with a velocity u from the position $s = s_0$, show that as time goes on, the particle approaches the position $s = s_0 + \frac{u}{K}$.

2. Answer any ***four*** of the following : *4×4=16*

(a) Evaluate any ***one*** of the following :

(i) $\int_0^{\pi/2} \log(\tan x)\, dx$

(ii) $\int \sin^5 x\, dx$

(b) Prove that :

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx = \frac{\pi}{4}$$

(c) Find the area of the region bounded by the parabola $y = x^2 + 2$ and the lines $y = x$, $x = 0$, and $x = 3$.

(d) Solve any ***one*** of the following :

(i) $(x + y)^2 \frac{dy}{dx} = 9$

(ii) $x\,dy - y\,dx = \sqrt{(x^2 + y^2)}\,dx$

(e) If $\frac{dv}{dt} = -\frac{v^2}{100}$ and $v = 15$ when $t = 0$, find the value of t when $v = 10$.

3. Answer any ***four*** of the following : *4×4=16*

(a) Let $\mathbf{A} = 2\hat{\mathbf{i}} + \hat{\mathbf{k}}$, $\mathbf{B} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, and $\mathbf{C} = 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$, determine a vector $\mathbf{R}$ satisfying

$\mathbf{R} \times \mathbf{B} = \mathbf{C} \times \mathbf{B}$ and $\mathbf{R} \cdot \mathbf{A} = 0$.

(b) A force $3\hat{\mathbf{i}} + \hat{\mathbf{k}}$ acts through the point $2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$. Find the torque about the point $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$.

(c) Determine the constant 'p' such that the vector $\mathbf{V} = (2x + 3y)\,\hat{\mathbf{i}} + (3y + 4z)\,\hat{\mathbf{j}} + (pz + 5x)\,\hat{\mathbf{k}}$ is solenoidal.

(d) A particle moves along a curve $x = t^3 + 3$, $y = t^2 + 4$, $z = 2t + 5$, where t is the time. Find the component of its velocity and acceleration at time $t = 2$ sec, in the direction $3\hat{i} + 4\hat{j} + 5\hat{k}$.

(e) A fluid motion is given by

$$\mathbf{V} = (y + z)\,\hat{i} + (z + x)\,\hat{j} + (x + y)\,\hat{k}.$$

Is the motion irrotational ? If so, find the velocity potential. Is the motion possible for an incompressible fluid ?

4. Answer any ***six*** of the following : *6×3=18*

(a) Prove that

$$\begin{vmatrix} -2a & a+b & c+a \\ a+b & -2b & b+c \\ c+a & b+c & -2c \end{vmatrix} = 4(b + c)(c + a)(a + b)$$

(b) Is the following a Skew – Hermitian matrix ?

$$\begin{bmatrix} 0 & 1+i & 3-2i \\ i-1 & 0 & 2+3i \\ -2i-3 & 3i-2 & 0 \end{bmatrix}$$

(c) Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

(d) Show that the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

satisfies the matrix equation

$$A^3 - 6A^2 + 5A + 11\,I = 0.$$

(e) Find the values of x for which the matrix A is singular when

$$A = \begin{bmatrix} 1-x & 2 & 2 \\ 2 & 2-x & 2 \\ 2 & 2 & 2+x \end{bmatrix}$$

(f) Solve the following equations by Cramer's rule or matrix method :

$$x - y + z = 4$$

$$2x + y - 3z = 0$$

$$x + y + z = 2$$

(g) Matrix A has x rows and x + 5 columns. Matrix B has y rows and 11 – y columns. Both AB and BA exist. Find x and y.

(h) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & \sqrt{3} \\ 0 & \sqrt{3} & 6 \end{bmatrix}$$