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No. of Printed Pages : 3

MMT-006

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

June, 2013

MMT-006 : FUNCTIONAL ANALYSIS

Time : 2 hours — *Maximum Marks : 50*

Weightage 70%

Note : *Question number* **1** *is* **compulsory.** *Attempt* **any four** *from the remaining questions.*

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1. State whether the following statements are **true** or **false**. Justify with a short proof or a counter example. **5x2=10**

(a) The space l^3 is a Hilbert space.

(b) Any non zero bounded linear functional on a Banach space is an open map.

(c) Every bounded linear map on a complex Banach space has an eigen value.

(d) The image of a Cauchy sequence under a bounded linear map is also a Couchy sequence.

(e) If A is a bounded linear operator on a Hilbert space such that $AA^* = I$, then $A^*A = I$.

2. (a) Characterise all bounded linear functionals on a Hilbert space. 5

(b) Prove that a normed linear space is complete if its unit sphere is complete. **3**

(c) Show that the map T : $\mathbf{R}^3 \to \mathbf{R}^2$ given by $T(x_1, x_2, x_3) = (x_1 + x_2, x_3)$ is an open map. **2**

3. (a) Show how a real linear functional u on a complex linear normed space gives rise to a complex linear functional f. What is the relation between the boundedness of u and that of f ? **4**

(b) For normal linear spaces X, Y, prove that BL (X, Y) is complete if Y is complete. **3**

(c) Prove that a finite dimensional normed linear space is always reflexive. **3**

4. (a) In a Hilbert space. Prove that $x_n \to x$ provided $\|x_n\| \to \|x\|$ and $\langle x_n, x\rangle \to \langle x, x\rangle$. **2**

(b) Are Hahn - Banach extensions always unique ? Justify. **3**

(c) State the principle of uniform boundedness. Use it to show that a set E in a normed space X is bounded if f(E) is bounded in K for every $f \in X'$. **5**

5. (a) If H is a Hilbert space and SCH, show that $S^{\perp} = S^{\perp\perp\perp}$. When S is the same as $S^{\perp\perp}$? Justify. **5**

(b) Show that Q defined on $(C[0, 1], \|\cdot\|_\infty)$ by $Q(x) = \int_0^1 t\, x(t)\, dt$ is a bounded linear functional. Calculate $\|Q\|$. 3

(c) Prove that l^∞ is not separable. 2

6. (a) Show that the dual of l^1 is isometrically isomorphic to l^∞. 6

(b) Let A be an operator on a Hilbert space H. Show that A is normal if and only if $\|Ax\| = \|A^*x\|$ for every $x \in H$. 4

7. (a) Let $\{u_n\}$ be the sequence in l^2 with 1 in the n^{th} place and zeroes else where prove that the set $\{u_n\}$ is an orthonormal basis for l^2. 3

(b) If X is a normed linear space and $0 \neq x_0 \in X$, show that there is a linear functional $f \in X'$ such that $f(x_0) = \|x_0\|$. 3

(c) Let E_1 be a closed subspace and E_2 a finite dimensional subspace of a normed linear space X. Show that $E_1 + E_2$ is closed in X. 4
