No. of Printed Pages : 3 **MMT-002**

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

June, 2013

MMT-002 : LINEAR ALGEBRA

Time : 1½ hours *Maximum Marks : 25*
(Weightage 70%)

Instruction : *Question No. **1** is **compulsory**. Do **any three*** questions from the rest. Use of Calculators ***not*** allowed.

---

1. Which of the following statements are **true** and which are **false** ? Justify your answer. **10**

(a) If the algebraic and geometric multiplicities of every eigen value of a square matrix A are equal, then the characteristic polynomial of A cannot have multiple roots.

(b) If A is a matrix which has a Moore - Penrose inverse, then A must be invertible.

(c) The sum of the eigen values of a matrix A is at most equal to the sum of the entries of A.

(d) A Hermitian matrix need not be a unitary matrix.

(e) The matrix A^*A is always positive definite, for any 2×3 matrix A.

2. (a) Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be a linear transformation defined by : 3

$$T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x + 2y + z \\ x + y + z \\ x + z \end{bmatrix}.$$

Find the matrix of T with respect to the basis

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

(b) Check whether the matrix 2

$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

is positive definite or not.

3. (a) Why is the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ similar to a diagonal matrix ? 2

(b) Find the quadratic polynomial which best fits the points $(-1, 10.7)$, $(2, 14)$, $(3, 27.9)$ and $(4, 48.2)$. 3

4. (a) Find the square root of $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$. 2

(b) Write the Jordan canonical form for a matrix A whose minimal polynomial is $(x-1)^2$ and the ranks of the matrices $A - I$ and $A-2I$ are 2 and 4, respectively. 3

5. Solve the system of differential equations 5

$$\frac{dy(t)}{dt} = Ay(t) \quad \text{with} \quad y(t) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{where}$$

$$A = \begin{bmatrix} 0 & 0 & 1 \\ -8 & 3 & 3 \\ -6 & 0 & 5 \end{bmatrix}$$