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B.Tech. MECHANICAL ENGINEERING / B.Tech. IN CIVIL ENGINEERING

Term-End Examination

June, 2013

BICE-027 : MATHEMATICS III

Time : 3 hours *Maximum Marks : 70*

*Note : Attempt **any ten** questions. All questions carry equal marks. Use of scientific calculator is **permitted**.*

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1. Prove that 7

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}, -\pi < x < \pi$$

Hence show that $\sum \frac{1}{n^2} = \frac{\pi^2}{6}$,

2. If $f(x) = \pi x \qquad 0 \le x \le 1$ 7

$\qquad = \pi(2-x) \qquad 1 \le x \le 2$

show that in the interval (0,2)

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left[\frac{\cos \pi x}{1^2} + \frac{\cos 3\pi x}{3^2} + \frac{\cos 5\pi x}{5^2} + \ldots\ldots\right]$$

Also deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$$

3. Express $f(x) = x$ as a half - range sine series in $0 < x < 2$. 7

4. Obtain a half - range cosine series for 7

$f(x) = kx$ for $0 \leq x \leq \frac{l}{2}$

$= k\,(l - x)$ for $\frac{1}{2} \leq x \leq l$

Also deduce the sum of the series

$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\cdots$

5. Solve : 7

$$(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial x}{\partial y} = ly - mx.$$

6. Solve : 7

$$\left(x^2 - y^2 - z^2\right)p + 2xyq = 2xz.$$

7. Solve : 7

$$\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 2\sin(3x + 2y)$$

8. Solve : $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$ 7

9. Solve : $\frac{dy}{dx} = (4x+y+1)^2$ 7

10. Solve : $\tan y \frac{dy}{dx} + \tan x = \cos y \cos^2 x$ 7

11. Solve : $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x} + \sin 2x.$ 7

12. The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What would be the value of N after 1½ hours ? 7

13. Use the method of separation of variable to solve 7

the equation $\frac{\partial^2 v}{\partial x^2} = \frac{\partial v}{\partial t}$

given that $v=0$, when $t \to \infty$

as well as $v=0$, at $x=0$, and $x=l$,

14. A rod of length l with insulated sides is initially at a uniform temperature u. Its ends are suddenly cooled to O°C and are kept at that temperature. Prove that the temperature function $u(x, t)$ is given by **7**

$$u(x,t)=\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} e^{\frac{-c^2\pi^2 n^2 t}{l^2}}$$

where b_n is determined from the equation

$$U_o=\sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$$

15. Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ **7**

Which satisfies the conditions

$u(0,y) = u(l,y) = u(x,0) = 0,$

and $u(x,a) = \sin\frac{n\pi x}{l}$.