BACHELOR OF TECHNOLOGY IN MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

Term-End Examination

June, 2013

02651

BME-001 : ENGINEERING MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are* ***compulsory.*** *Use of statistical tables and calculator is* ***permitted.***

1. Answer *any five* of the following : 5x4=20

(a) Find $\frac{dy}{dx}$ where $y = \tan^{-1} \frac{\sqrt{1 + x^2} - 1}{x}$

(b) Evaluate any one of the following :

(i) $\lim\limits_{x \to 0} \frac{x - \sin x}{x^3}$

(ii) $\lim\limits_{x \to 2} \left[\frac{4}{x^2 - 4} - \frac{1}{x - 2} \right]$

(c) Show that the tangent at (a, b) to the curve

$$\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 = 2 \text{ is } \frac{x}{a} + \frac{y}{b} = 2$$

(d) Evaluate (***any one***)

(i) $\int \frac{x}{1 + \cos x} dx$

(ii) $\int \frac{\cos x}{\sin x + \cos x} dx$

(e) Given $\frac{dv}{dt} = -\frac{v^2}{100}$ and $v = 15$ when $t = 0$,

find the value of t when $v = 10$.

(f) If $u = x + y + z$, $uv = y + z$, $uvw = z$,

show that $\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2 v$

2. Answer *any four* of the following : **4x4=16**

(a) Show that

$$V = (y + \sin z)\hat{i} + x\hat{j} + x \cos z \hat{k}$$

is a conservative vector field. Find $\phi(x,y,z)$, such that $V = \text{grad } \phi$.

(b) Find the projection of the vector

$A = \hat{i} - 2\hat{j} + \hat{k}$, on the vector

$B = 4\hat{i} - 4\hat{j} + 7\hat{k}$,

(c) Find the work done in moving an object along a straight line from $(3, 2, -1)$ to $(2, -1, 4)$ in a force field given by

$$F = 4\hat{i} - 3\hat{j} + 2\hat{k},$$

(d) If $A = x^2 y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$, find curl (curl A).

(e) Determine the constant a so that the vector

$$V = (x + 3y)\hat{i} + (y - 2z)\hat{j} + (x + az)\hat{k}$$

is solenoidal.

(f) Verify Green's theorem in the plane for

$\oint_c \left[xy + y^2\right]dx + x^2\,dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

3. Answer *any six* of the following : **6x3=18**

(a) Prove that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

(c) The mean life time of a sample of 100 fluorescent light bulbs produced by a company is computed to be 1570 hours with a standard deviation of 120 hours. The company claims that the average life of the bulbs produced by it is 1600 hours. Using the level of significance of 0.05, is the claim acceptable ?

(d) In a certain factory turning out razor blades, there is a small chances of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate for approximate number of packets containing no defective, one detective and two defective blades respectively in a consignment of 10000 packets.

(e) The means of simple samples of sizes 1000 and 2000 are 67.5 cm and 68.0 cm respectively. Can the samples be regarded as drawn from the same population of SD 2.5 cm.

(f) A machinist is making engine parts with axle diameter of 0.7 inch. A random sample of 10 parts shows mean diameter 0.742 inch with a standard deviation of 0.04 inch. On the basis of this sample, would you say that the work is inferior at 5% level of significance.

(b) If $A = \begin{bmatrix} 5 & 4 & -2 \\ 4 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix}$

show that $A^2 - 11A + 10\ I = 0$

(c) Where I and O are the unit and zero matrix of order 3 respectively.

If $\begin{bmatrix} x-z & -x-z \\ 7-t & 6+z \end{bmatrix} = \begin{bmatrix} 3-t & 5-t \\ t+5 & x-y \end{bmatrix}$

Find the values of x, y, z and t.

(d) Obtain the rank of the matrix

$$A = \begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$$

(e) If $A = \begin{bmatrix} -1 & 2+i & 5-3i \\ 2-\hat{i} & 7 & 5i \\ 5+3i & -5i & 2 \end{bmatrix}$,

Show that A is a Hermitian matrix and i A is a skew-Hermitian matrix.

(f) Find the eigen values of the matrix $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.

(g) Solve by Cramer's rule

$$x + 2y - z = 9$$

$$2x - y + 3z = -2$$

$$3x + 2y + 3z = 9$$

(h) Express the matrix A as the sum of a symmetric and a skew-symmetric matrix

$$\text{where } A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}$$

4. Answer *any four* of the following : **4x4=16**

(a) One bag contains 4 white balls and 3 black balls, and a second bag contains 3 white balls and 5 black balls. One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from the second bag is black ?

(b) A lot contains 20 articles. The probability that the lot contains exactly 2 defective articles is 0.4 and that it contains exactly 3 defective articles is 0.6. Articles are drawn from the lot at random one by one without replacement and tested till all the defective articles are found. What is the probability that this procedure ends at the twelfth testing ?