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ET-102

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering)

Term-End Examination 01614

June, 2013

ET-102 : MATHEMATICS - III

Time : 3 hours *Maximum Marks : 70*

Note : *Question* ***No.1*** *is* ***compulsory****. Attempt any other* ***eight*** *questions from* ***q. no. 2*** *to* ***q. no. 15****. Use of calculator is* ***allowed****.*

1. Complete the following : **7x2=14**

(a) The sequence $<x_n>$, where $x_n = nt\, e^{-nt^2}$, is not uniformly convergent on (0, 1) and attains maximum value _________ at t= _________ .

(b) If L^{-1} denotes Laplace Inverse, then

$$L^{-1}\left[\frac{1}{(s-1)(s-2)}\right] = \_\_\_\_\_\_\_\_\_ .$$

(c) If the series $\sum an$ is convergent, then the series $\sum an \cdot \frac{x^n}{1+x^{2n}}$ converges uniformly in _________ .

(d) The analytic function $f(z) = w = u + iv$ for which $u - v = (x - y)(x^2 + 4xy + y^2)$ is ________.

(e) If C is the circle $|z| = 3$, then

$$\int_c \frac{z+2}{(z+1)^2 (z-2)} dz = \_\_\_\_\_\_\_\_ .$$

(f) If $p_n(x)$ is a Legendary polynomial, then $P'_n(-x)$ in terms of $p_n(x)$ and its derivatives is ________ .

(g) The partial differential equation : $(D - D')^2 u = e^{x+2y}$ has the particular integral as ________ .

2. (a) Apply Picord's method to find first two approximations to the solution of IVP **3½**

$$\frac{dy}{dx} = 2y - 2x^2 - 3 \text{ with } y(0) = 0$$

(b) Solve $(2x - 10y^2)\, dy + y dx = 0$ **3½**

3. Find the power series solution about $x = 1$ of the initial value problem **7**

$$x \frac{d^2y}{dx} + \frac{dy}{dx} + y = 0 \text{ with } y(1) = 1, \left.\frac{dy}{dx}\right|_{(y=1)} = 2$$

4. Find the equation of integral surface of the p.d.e. 7

$(xy^3 - 2x^4)p + (2y^4 - x^3y)q = 9z(x^3 - y^3)$.

5. Using Laplace Transforms, solve 7

$(D^3 - 1)y = e^t$, with $y(0) = 0$, $y'(0)$, $y''(0) = 0$

6. (a) Show that 3½

$$L\,(\log t) = \frac{I'(1) - \log s}{s}$$

(b) Using First Shifting Theorem, find 3½

$$L^{-1}\left(\frac{1}{s^2 - 4s + 20}\right).$$

7. Test the sequence $\langle a_n \rangle$, defined by the relation 7

$$a_n = 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \text{-----} \frac{1}{\lfloor n-1},$$

for bounded, monotonocity and convergence.

8. Test the convergence of the series 2x3½

(a) $1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \text{-----}$

(b) $1 - \frac{1}{3.2} + \frac{1}{3^2.3} + \frac{1}{3^3.4} + \text{-----}.$

9. Find half-range cosine series for the function 7

$$f(x) = \begin{cases} x & \text{for } 0 < x < \frac{\pi}{2} \\ \pi - x & \text{for } \frac{\pi}{2} < x < \pi \end{cases}$$

10. Find the Fourier series generated by periodic function $|x|$ of period 2π. Also compute the value of series at -3π. 7

11. (a) Find the characteristic function, transfer function and frequency response function for the equation $(D+4D^{-1})x = f(t)$ 3

(b) Test the following differential equation for stability : $(D^3+1)x = f(t)$ 4

12. Apply tabular form of Harwitz-Routh criterion to test the stability of the differential equation $(D^4+7D^3+17D^2+17D+6)\ y = f(x)$. 7

13. Using the method of separation of variables, find the solution of the heat conduction problem 7

$u_{xx} = 4u_{tt},\ 0 < x < 2,\ t > 0,$

$u(0, t) = 0 = u\ (2, t),\ t \geqslant 0,$

$u(x, 0) = 2\sin\frac{\pi x}{2} - \sin \pi x + 4\sin 2\pi x.$

14. (a) Find the value of $\int_{c:|z|=1} e^{2z}(z+1)^{-2}\,dz$ 3½

(b) Find the Laurent's expansion of the function 3½

$$f(z) = \frac{7z-2}{(z+1)\,z(z-2)} \text{ in the annulas } 0 < |z+1| < 1.$$

15. Using the method of complex integration, evaluate 7

$$\int_0^{\pi} \frac{a\,d\theta}{a^2+\sin^2\theta}, \quad a > 0.$$