No. of Printed Pages : **4**

MCS-031

11604

MCA (Revised)

Term-End Examination

June, 2013

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours *Maximum Marks : 100*

Note : *Question* ***No. 1*** *is* ***compulsory****. Attempt* ***any three*** *from the rest of questions.*

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1. (a) (i) Write an algorithm to sort the given array of numbers using Insertion Sort and explain the necessary steps. Also write the time complexity in the worst case. 6

(ii) Prove that function : 6

$$f(x) = 2x^3 + x^2/4 + 100 \text{ is } \theta(x^3).$$

(b) (i) Solve the following recurrence relation : 5

$$f_n - f_{n-1} - f_{n-2} = 0 \text{ s.t. } f_0 = 0, \; f_1 = 1$$

(ii) Prove that the running time of binary search algorithm in worst case is $0(\log_2 n)$. 5

(c) (i) Define undecidability. List any three undecidable problem. 5

(ii) Discuss, why do we use dynamic programming approach to solve matrix chain multiplication problem. 5

(d) (i) There are five bottles of medicine, namely A, B, C, D and E. The capacity of each bottle in term of number of tablets it can hold is given below : 5

Bottle A : 10 tablets, Bottle B : 60 tables,

Bottle C : 130 tablets,
Bottle D : 240 tablets, Bottle E : 100 tablets.

Give a greedy approach to store 560 tablets by using minimum number of bottles.

(ii) Analyse the time complexity for the following : 3

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for (i=1 ; i≤n ; i=i*2)
for (j=1 ; j≤n ; j++)
Count=count+2 ;
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2. (a) If L_1 and L_2 are regular languages. Prove that $L_1 \cup L_2$ is also regular. 5

(b) Write a recursive function to calculate the sum of all elements in an integer array. 5

(c) Define a Context-Free Grammar (CFG). Write a CFG for the following language. **10**

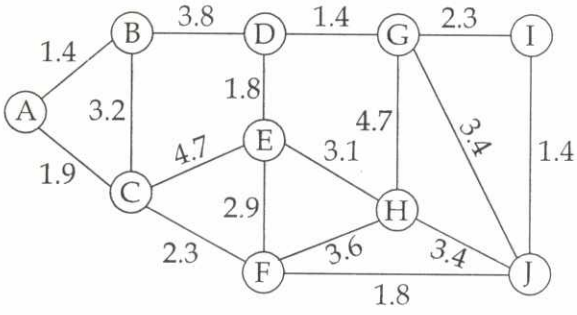
(i) $L = \{a^m b^n : m > n, m, n \geq 0\}$

(ii) $L = \{\omega\omega^R : \omega \epsilon \{a, b\}^*\}$ where ω^R is a reverse of ω.

3. (a) What is "principle of optimality" in Dynamic Programming ? Explain how dynamic programming can be used to solve a chain of matrix multiplication ? Apply Dynamic Programming to multiply the following (4) matrices. $<M_1, M_2, M_3, M_4>$ with dimensions <(15, 3), (3, 8), (8, 9), (9, 7)>. **10**

(b) Write an algorithm for Quick Sort. Analyse the Running time of Quick Sort in Best and worst case. **10**

4. (a) Use Kruskal's algorithm to construct a minimum spanning tree from the following graph. **5**



(b) Define O/I Knapsack Problem. Using Dynamic programming. Solve the following O/I Knapsack problem. 10

No. of objects n = 5,

Capacity of Knapsack, M = 16

Weight	2	3	6	5	9
Profit	2	7	15	20	12

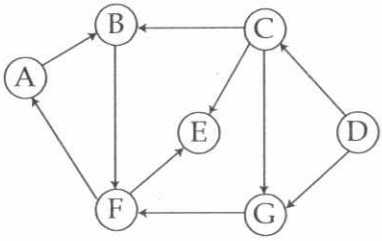
(c) Write an algorithm for Topological Sort. 5

5. (a) Sort the following elements using Heap Sort : 7

47, 29, 82, 11, 48, 32, 28, 17, 65, 36

Show each step, while creating a heap and processing a heap.

(b) Use breadth first search to traverse the following graph by using C as the starting node. 5



(c) Write an algorithm to multiply given two n-bit decimal number $X = x_{n-1}\, x_{n-2} \ldots x_1\, x_0$ and $Y = y_{n-1}\, y_{n-2} \ldots y_0$, using Divide and Conguer method. Also analyse the time complexity of the algorithm used ? 8