No. of Printed Pages : **11** **MIA-009 (F2F)**

M.Sc. ACTUARIAL SCIENCE

Term-End Examination

June, 2012

MIA-009 (F2F) : GENERAL INSURANCE, LIFE AND HEALTH CONTINGENCIES

Time : 3 hours *Maximum Marks : 100*

Note : *In addition to this paper you should have available Actuarial Table and your own electronic calculator.*

SECTION - A

(Answer ***any five*** questions)

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1. Let X be a random variable representing the present value of the benefits of a whole of life assurance, and Y be a random variable representing the present value of the benefits of a temporary assurance with a term of n-years. Both assurances have a sum assured of 1 payable at the end of the year of death and were issued to the same life aged X.

(a) Describe the benefits provided by the contract which has a present value represented by the random variable X - Y. **1**

(b) Show that 7

$$\text{Cov}(x, y) = {}^2A^{\,1}_{x:\overline{n}|} - A_x\, A^{\,1}_{x:\overline{n}|}$$

and hence or ortherwise that :

$$\text{Var}(x-y) = {}^2A_x - (n|A_x)^2 - {}^2A^{\,1}_{x:\overline{n}|}$$

Where the functions A are determined using an interest rate of i, and functions ^2A are determine using an interest rate of i^2+2i.

2. (a) Show that 4

$${}_{t-s}q_{x+s} = \frac{{}_{(t-s)}q_x}{(1-{}_s q_x)} \quad (0 \le s < t \le 1)$$

using an assumption of a uniform distribution of deaths.

(b) Calculate the value of ${}_{0.5}q_{62.25}$ using assumption of 4

(i) a uniform distribution of death

(ii) a constant force of mortality

Basis : Mortality PMA92C20

3. (a) An annuity payable continuously through out the lifetime of a pension now aged exactly 60, but for at most 10 years. The rate of payment at all times t during the first 5 years is ₹ 10000 pa, and thereafter it is ₹ 12000 pa. The force of mortality of this life is 0.03 pa between the ages of 60 and 65 and 0.04 pa between the ages of 65 and 70. Calculate the expected present value of this annuity assuming a force of interest of 0.05 pa. 5

(b) Assuming that mortality and interest are as in (i) above, calculate the expected present value of a 10 - year term assurance issued to the life in (i), which pays ₹ 50000 immediately on death. **3**

4. An n - year term assurance with a sumassured of 1 payable at the end of the year of death is issued to a life aged x. Level premiums are payable annually in advance throughout the term of the policy or until the policyholder's earlier death. The premium includes an initial expenses loading of I, and a renewal expense loading of e at the start of each policy year, including the first.

(a) Give expression in terms of standard actuarial functions, for : **3**

(i) the gross premium

(ii) the prospective gross premium reserve at (integer) time $t < n$

(iii) the retrospective gross premium resene, at (integer) time $t < n$

(b) Hence show that, if all three of the expression in (i) are calculated on the same basis, the prospective and retrospective gross premium reserve are equal. **5**

5. You are using three - state illness - death model to price verious sickness policies. Using the actuarial notation, write down an expression for the expected present value of each of the following sickness benefits for a healthy life aged 30.

(a) ₹ 3000 pa payable continuously while ill, but ceasing at age 60. **2**

(b) ₹ 3000 pa payable continuously through out the first period of illness only, but ceasing at age 60. **3**

(c) ₹ 3000 pa payable continuously while ill provided that the life has been ill for at least one year. Again any benefits ceases to be paid at age 60. **3**

6. A life insurance company sells whole - life assurance policies with a sum assured of ₹ 20000 payable at the end of the year of death. The premium of ₹ 420 payable annually in advance until the death of the policyholder.

A life now aged 50 purchased a policy exactly one year ago, and is now due to pay the second annual premium.

(a) Find the expected present value of the future loss to the company arrising from this policy. **3**

(b) Show that the variance of the present value of the future loss from this policy can be expressed as **5**

b. A'_{50} + C

Determine the numerical value of b and c, and the rate of interest used to evaluate

A'_{50}.

Basis : mortality AM 92 ultimate

4% pa interest

ignore expenses

7. Prove Thieles differential equation for an endowment assurance issued to a life aged x to be as follows. **8**

$$\frac{\partial}{\partial t}\left[{}_t\bar{V}_{x:\overline{n|}}\right] = -\left(1 - {}_t\bar{V}_{x:\overline{n|}}\right)\mu_{x+t} + \delta\, {}_t\bar{V}_{x:\overline{n|}} + \bar{P}_{x:\overline{n|}}$$

SECTION-B

(Answer *any four* questions)

8. On 1 January 2001 an insurer issued a block of 25 -year annual premium endowment policies that pay ₹ 120000 at maturity, or ₹ 60000 at the end of earlier death to lives aged exactly 65. The premium basis assumed 4% interest, AM 92 select mortality and allowed for an initial expenses of ₹ 200 and renewal expenses of 1% of each subsequent premium. Reserve are calculated on the same basis as the premiums.

   (a) Calculate the premium. **4**

   (b) Calculate the reserve required per policy at 31 December 2005 **4**

   (c) There were 197 policies in force on 1 January 2005. During 2005 there were 9 deaths, interest was earned at the twice the rate expected and expenses were incurred at twice the expected. By considering the total reserve required at the start and end of the year, and all the cashflows during the year, calculate the profit or loss made by the insurer from all sources (not just from mortality) in respect of these policies for the 2005 calender year. **7**

9. On 1 May 1998, a life insurance company issued a whole life with - profits policy to a life then aged exactly 45. The basic sum assured was ₹ 50000. The sum assured and attaching bonuses are payable 3 months after the death of the policyholder. Level monthly premiums are payable in advance for the whole of life. The company calculated the premium on the following basis

Mortality	:	AM 92 select
Interest	:	6% pa
Bonus loading	:	1.9231% pa compound, vesting at the end of each policy year
Expenses	:	Initial ₹ 300 renewal 5% of each premium, excluding the first.
Termination	:	₹ 200 payable at the same time as the death benefit.

(a) Show that the monthly premium is ₹ 85.65. 7

The company hold gross premium retrospective reserves for the policy, calculated on the following basis

Mortality	:	AM 92 select
Interest	:	4% pa
Past bonuses	:	4% pa compound, vesting at the end of each policy year

Expenses : Initial : ₹ 300

renewal : ₹ 5 at the start of each month, excluding the first

termination : ₹ 100 payable at the same time as the death benefit.

(b) Calculate the reserve for the policy on 30 April 2005. 8

10. (a) A joint life annuity of 1 pa is payable continuously to lives currently aged x and y while both lives are alive. The present value of the annuity payments is expressed as a random variable, in terms of the joint future life time of x and y. 6

Derive and simplify as far as possible expressions for the expected present value and variance of the present value of the annuity.

(b) Ralph and Ted are both aged 60 exact and their mortality follows PMA92C20. Upon Ted's death, Ralph will receive ₹ 20000 pa payable annually in advance for the rest of his life, starting from the end of the year of Ted's death. The payments to Ralph will continue for 12 years after Ralph has died. No payments are made if Ralph dies first. The interest rate for all future years is $i = 4\%$ pa. You are given that $A_{60:60} = 0.47585$ where both lives follow PMA92C20. Calculate the EPV of this benefit to Ralph. 9

11. A life insurance company issues a number of 3 - year term assurance contracts to lives aged exactly 60. The sum assured under each contract is ₹ 200000 payable at the end of the year of death. Premiums are payable annually in advance for the term of the policy, ceasing on earlier death. The company carries out profit tests for these contracts using the following assumptions.

Initial expenses	: Rs 200 plus 35% of the first year's premium
Renewal expenses	: Rs 25 plus 3% of the annual premium incurred at the beginning of the second and subsequent years.
Mortality	: AM 92 Ultimate
Investment return	: 7% per annum
Risk discount rate	: 15% per annum
Reserves	: One year's office premium

(a) Show that the office premium, to the nearest pound, is ₹ 2527, if the net present value of the profit is 25% of the office premium. **10**

(b) Calculate the cash flows if the company held zero reserves throughout the contract, using the premium calculated in part (a) **2**

(c) Explain why the company might not hold reserve for the contract and the impact on profit if they didnot hold any reserves. **3**

12. A pension scheme provides a pension of 1/60 of career average salary in respect of each full year of service, on age retirement between the ages of 60 and 65. A proportionate amount is provided in respect of an incomplete year of service. 15

At the valuation date of the scheme a new member aged exactly 40 has an annual rate of salary of ₹ 40000.

Define all your terms and stating any assumptions made, calculate the expected present value of the future service pension on age retirement in respect of this member, using the pension fund table in the formula and table for actuarial examination.

13. (a) A life insurance company has renewed its mortality experience. For each age it has pooled all the deaths and corresponding exposures from its entire portfolio over the previous ten years, and derived a single mortality table. 9

List three types of selection which might be likely to produce heterogeneity in this particular investigation. In each case explain the nature of the heterogeneity and how it could be caused, and state the heterogeneity can be reduced.

(b) The company has produced the following data in respect of two locations. Calculate the standardised mortality ratio for each location based on the standard mortality table ELT 15 (males). 6

Age	Location A		Location B	
	Initial exposed to risk	Number of death	Initial exposed to risk	Number of death
60	100	1	200	3
61	175	3	150	3
62	190	2	170	3
63	210	3	100	2