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**MIA-005 (F2F)**

# M.Sc. IN ACTUARIAL SCIENCE (MSCAS)

## Term-End Examination

## June, 2012

## MIA-005 (F2F) : STOCHASTIC MODELLING AND SURVIVAL MODELS

*Time : 3 hours* *Maximum Marks : 100*

**Note :** *In addition to this paper you should have available the ACTUARIAL table and your own electronic calculator.*

### SECTION-A

(Answer ***any five*** questions)

**1.** List the benefits and limitations of modelling in actuarial work. **8**

**2.** For a stochastic process Xn with time space J and state space S, define the term :

(a) (i) Stationary **1**

(ii) Weakly Stationary **1**

(iii) Increment **1**

(iv) Markov property **1**

(b) (i) Define a Poisson process with rate $\lambda$. **1**

(ii) Define a compound Poisson process. **1**

(iii) Explain why the compound poisson process has the markov property ? **2**

5. A life insurance company prices its long-term sickness policies using a three - state Markov model in continuous time. The states are healthy (H), ill (I) and dead (D). The force of transition in the model are $\sigma_{HI} = \sigma$, $\sigma_{IH} = \rho, \sigma_{HD} = \mu, \sigma_{ID} = \nu$ and they are assumed to constant over time.

For a group of policyholders observed over a 1 - year period, there are :

23 transitions from state H to State I ;

15 transitions from state I to state H ;

3 deaths from state H ;

5 death from state I.

The total time spent in state H is 652 years and the total time spent in state I is 44 years.

(a) Write down the likelihood function for these data. **4**

(b) Derive the maximum likelihood estimate of $\sigma$. **2**

(c) Estimate the standard deviation of $\tilde{\sigma}$, the maximum likelihood estimator of $\sigma$. **2**

6. A No - claim Discount system operated by a motor insurer has the following four levels.

- Level 1 : 0% discount
- Level 2 : 25% discount
- Level 3 : 40% discount
- Level 4 : 60% discount

The rules for moving between these levels are as follows :

- Following a year with no claims, move to the next higher level, or remain at level 4.
- Following a year with one claims, move to the next lower level, or remain at level 1.
- Following a year with two or more claims, move down two levels, or move to level 1 (from level 2) or remain at level 1.

For a given policyholder in a given year the probability of no claims is 0.85 and the probability of making one claim is 0.12.

(a) Write down the transition matrix of this No-Claims Discount process. **1**

(b) Calculate the probability that a policyholder who is currently at level 2 will be at level 2 after two years. **2**

(c) Calculate the long-run probability that a policyholder is in discount level 2. **5**

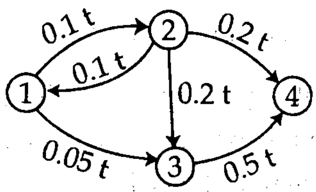
7. (a) Explain the term "undergraduation" and "overgraduation". **3**

(b) List the possible dangers to a life company of using undergraduated or overgraduated mortality rates. **5**

## SECTION-B

(Answer *any four* questions)

8. Consider the following time - inhomogeneous Markov jump process with transition rates as shown below :




(a) Write down the generator matrix at time t. **2**

(b) Write down the Kolmogorov backward differential equations for $P_{33}(s,t)$ and $P_{13}(s,t)$. **3**

(c) Using the technique of separation of variables, or otherwise, show that the solution of the differential equation for $P_{33}(s,t)$ is : **4**

$$P_{33}(s,t) = \exp[-0.25(t^2 - s^2)]$$

(d) Show that the probability that the process visits neither state 2 nor state 4 by time t, given that it starts in state 1 at time 0, is : **6**

$$\frac{8}{7}e^{-0.075\,t^2} - \frac{1}{7}e^{-0.25t^2}$$

9. (a) Show that if the force of mortality $\mu_{x+t}$ $(0 \leq t \leq 1)$ is given by : 4

$$\mu_{x+t} = \frac{q_x}{1+t.q_x}$$

this implies that deaths between exact ages $x$ and $x+1$ are uniformly distributed.

(b) Studies of the lifetimes of a certain type of electric light bulb have shown that the probability of failure, $q_0$, during the first day of use is 0.05 and after the first day of use the " force of failure". $\mu_x$, is constant at 0.01.

(i) Calculate the probability that a light bulb will fail within the first 20 days. 2

(ii) Calculate the complete expectation of life (in days) of : 7

(A) a one-day old light bulb.

(B) a new light bulb.

(iii) Comment on the difference between the complete expectations of life calculated in (ii) (A) and (B). 2

10. A life insurance company has carried out a mortality investigation. It followed a sample of independent policyholders aged between 50 and 55 years. Policyholders were followed from their $50^{th}$ birthday until they died, withdrew from the investigation while still alive, or reached their $55^{th}$ birthday (whichever of these events occured first).

(a) Describe the types of censoring that are present in this investigation. **3**

(b) An extract from the data for 12 policyholders is shown in the table below. Use these data to calculate the Nelson - Aalen estimate of the survival function. **5**

| Life | Last age at which life was observed | Reason for exit | |
|---|---|---|---|
| | Years | Month | |
| 1 | 50 | 9 | Died |
| 2 | 51 | 3 | Withdrew |
| 3 | 51 | 6 | Died |
| 4 | 51 | 6 | Died |
| 5 | 51 | 6 | Withdrew |
| 6 | 52 | 9 | Withdrew |
| 7 | 53 | 3 | Withdrew |
| 8 | 54 | 3 | Died |
| 9 | 54 | 6 | Died |
| 10 | 55 | 0 | Reached age 55 |
| 11 | 55 | 0 | Reached age 55 |
| 12 | 55 | 0 | Reached age 55 |

(c) Determine an approximate 95% confidence interval for your estimate of the survival function. **7**

11. The following model for the force of mortality for a life insurance company's annuitants has been proposed :

$$\mu(t,i) = (0.015 - 0.0001t) \exp [\alpha(x_i - 70) + \beta y_i + \gamma z_i]$$

where

$\mu(t,i)$ = force of mortality for the ith life in calendar year 2000 + t.

$x_i$ = age of the ith life

$$y_i = \begin{cases} 1 & \text{life is smoker} \\ 0 & \text{life is non - smoker} \end{cases}$$

$$zv = \begin{cases} 1 & \text{life is male} \\ 0 & \text{life is female} \end{cases}$$

$\alpha$, $\beta$, $\gamma$ are the parameter of the model.

The following data have been observed over the calendar year 2003 :

| Risk characteristic | Number of annuitant | Number dying |
|---|---|---|
| Male non - smoker, average age 65 | 800 | 6 |
| Male smoker average age 60 | 200 | 5 |
| Female non - smoker, average age 70 | 450 | 2 |
| Female Smoker, average age 65 | 150 | 1 |

You can assume that the number of annuitants in each class remained constant throughout the investigation period, and that the average age for each class can be treated as representing the value of $x_i$ for each individual in that class.

(a) Explain why this model is a proportional hazards model. **2**

(b) Explain the importance of subdividing the data by age, sex and smoking status, and explain whether you think each of the parameters $\alpha$, $\beta$ and $\gamma$ would be likely to be positive or negative. **4**

(c) Calculate the force of mortality for female non smoker with average age 70 in 2007, according to this model. **1**

(d) (i) Obtain an expression for the partial likelihood based on the given data expressing your answer in terms of $\alpha$, $\beta$ and $\gamma$ only. **8**

(ii) State how you would estimate the parameters of the model using the partial likelihood.

**12.** (a) Describe the circumstances under which it would be appropriate to graduate the rates in a mortality investigation using a mathematical function. **5**

(b) A graduation of a set of assured lives mortality data has been carried out and you are given the following results : **10**

| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---|---|---|---|---|---|---|
| Age $x$ | Initial Exposed to risk $E_x$ | Actual deaths $\theta_x$ | Graduated mortality rate $q_x$ | Expected death $E_x\, q_x$ | Standard deviation $\sqrt{E_x \hat{q}_x (1-\hat{q}_x)}$ | Standardised Deviation $\frac{(3)-(5)}{(6)}$ |
| 50 | 35000 | 161 | 0.004810 | 168.3 | 12.94 | −0.5641 |
| 51 | 40000 | 205 | 0.005340 | 213.6 | 14.58 | −0.5898 |
| 52 | 45000 | 260 | 0.005928 | 266.8 | 16.28 | −0.4177 |
| 53 | 50000 | 344 | 0.006579 | 329.0 | 18.08 | 0.8296 |
| 54 | 55000 | 418 | 0.007300 | 401.5 | 19.96 | 0.8267 |
| 55 | 60000 | 506 | 0.008097 | 485.8 | 21.95 | 0.9203 |
| 56 | 50000 | 463 | 0.008977 | 448.8 | 21.09 | 0.6733 |
| 57 | 40000 | 388 | 0.009948 | 397.9 | 19.85 | −0.4987 |
| 58 | 30000 | 318 | 0.011017 | 330.5 | 18.08 | −0.6914 |
| 59 | 25000 | 296 | 0.012194 | 304.8 | 17.35 | −0.5072 |
| 60 | 20000 | 251 | 0.013489 | 269.8 | 16.31 | −1.1527 |
| Total | 450,000 | 3610 | | 3616.8 | | |

Carry out a serial correlation test (at lag 1) on these data, and state your conclusion.

13. A large life office is investigating the recent mortality experience of its term assurance policyholders. It has been decided to graduate the data by reference to a standard table using the

formula $\dfrac{q_x}{q_x^s} = a\,x + b$

where $q_x^s$ is the rate for the standard table.

(a) Outline the rate considerations that you would take into account in choosing an appropriate standard table. 4

(b) Explain how you would check whether the above formula was suitable ? 3

(c) Describe briefly how you would estimate a and b in the formula using ? 8

(i) a weighted least square method.

(ii) a maximum likelihood method.