MMTE-005

00113

M.Sc. MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE (MACS)

Term-End Examination

June, 2012

MMTE-005 : CODING THEORY

Time : 2 hours *Maximum Marks : 50*

Note : *Do **any five** questions from question 1 to 6. Use of Calculator is **not** allowed.*

1. (a) Let C be the binary linear block code having generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(i) Find all the code words of C. **3**

(ii) Find parity check matrix of C. **1**

(iii) Find all the code words of the dual code of C. **3**

(b) Define convolutional code and give an example of convolutional code. **3**

2. (a) Let C be the binary code with the following generator matrix : **4**

$$G = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Find the weight distribution of C and hence find the weight enumerator polynomial.

(b) Let C be the binary code with generator matrix. **6**

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Decode the following received words :

(i) [1, 1, 0, 1, 0, 1, 1]

(ii) [0, 1, 1, 0, 1, 1, 1]

(iii) [0, 1, 1, 1, 0, 0, 0]

3. (a) State the two way APP decoding algorithm. **3**

(b) Given that β is the primitive element and x^3+2x+1 is its minimal polynomial in $\frac{F_3\,[x]}{\langle\, x^3+2x+1 \,\rangle}$, minimal polynomial of β^2 is x^3+x^2+x+2 and the minimal polynomial of β^4 is x^3+x^2+2, construct a ternary BCH code of length 26 and design distance 5. **4**

(c) Make the multiplication table for the finite field $\frac{F_2\,[x]}{\langle\, x^2+x+1 \,\rangle}$. **3**

4. (a) Find all irreducible polynomials of degree 1, 2, 3 and 4 over $\mathbf{F}_2$. **6**

(b) Write all the possible generator polynomials of a [7,4] cyclic code. Obtain the generator matrix and parity check matrix corresponding to any one of the generating polynomials. **4**

5. (a) If $f(x) \epsilon Z_4[x]$ is a basic irreducible polynomial show that $f(x)$ is a primary polynomial. **4**

(b) Prove that duadic codes of length n over F_q exist if and only if q is a square moduls n. **6**

6. (a) Let C be the (16, 3, 4) LDPC code with the parity check matrix given below : **6**

$$
\left[\begin{array}{cccc|cccc|cccc|cccc}
1 & 1 & 1 & 1 & & & & & & & & & & & & \\
 & & & & 1 & 1 & 1 & 1 & & & & & & & & \\
 & & & & & & & & 1 & 1 & 1 & 1 & & & & \\
 & & & & & & & & & & & & 1 & 1 & 1 & 1 \\
\hline
1 & & & & 1 & & & & 1 & & & & 1 & & & \\
 & 1 & & & & 1 & & & & 1 & & & & 1 & & \\
 & & 1 & & & & 1 & & & & 1 & & & & 1 & \\
 & & & 1 & & & & 1 & & & & 1 & & & & 1 \\
\hline
1 & & & & & 1 & & & & & 1 & & & & & 1 \\
 & 1 & & & & & 1 & & & & & 1 & 1 & & & \\
 & & 1 & & & & & 1 & 1 & & & & & 1 & & \\
 & & & 1 & 1 & & & & & 1 & & & & & 1 &
\end{array}\right]
$$

Decode the code word [1000010110100100] using hallagher hard decision decoding algorithm.

(b) Find the 3 - cyclotomic cosets moduls 9. 2

(c) Define a Reed-Solomon code. Also give an example of a Reed Solomon code. 2
