No. of Printed Pages : 5

MMTE-001

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) (MACS)
M.Sc. (MACS)

Term-End Examination

June, 2012

00613

MMTE-001 : GRAPH THEORY

Time : 2 hours *Maximum Marks : 50*

*Note : **Question** No. 1. is compulsory. Answer **any four** from the remaining **six** (2-7). Calculators and similar devices are not allowed.*

1. Are the following statements is true or false ? Give reasons for your answers. **10**

(a) If G is isomorphic to H, then the complement of G is isomorphic to the complement of H.

(b) If the minimum degree $\delta(G) \geq 2$, then G contains a cycle.

(c) If G has a spanning tree, then G is connected.

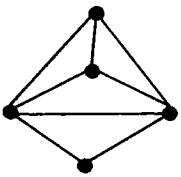
(d) If $\chi(G) = n$, then G contains K_n as a subgraph.

(e) Every Eulerian graph is 2 - connected.

2. (a) Draw a graph G with. **4**

rad (G) < diam (G) < 2 rad (G).

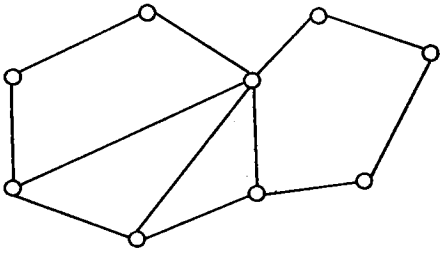
(b) Draw the dual of the following graph : 3



(c) For the following graph G find the following : 3

(i) $\kappa'(G)\ \kappa'(G)$

(ii) A separating set S with $|S| = \kappa$



3. (a) Draw a graph with six vertices and 9 edges which is 4

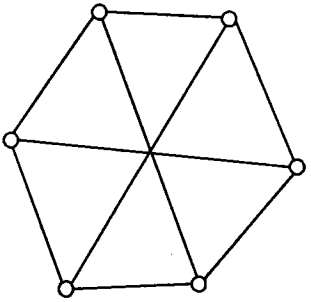
(i) Planar (ii) Non planar

(b) Draw a graph with n vertices, $n-1$ edges but having a cycle. 2

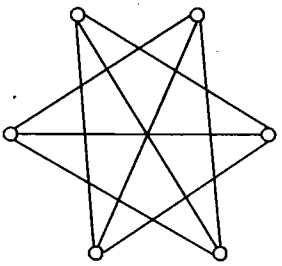
(c) Use Havel-Hakimi theorem to check whether the sequence $\{5, 5, 4, 4, 3, 3, 2, 2,\}$ is graphic or not. If the sequence is graphical, construct a graph with the above degree sequence. 4

4. (a) State with justification, whether the following graphs are isomorphic or not. 3

G :

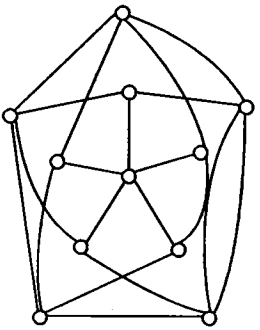


H :

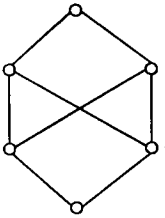


(b) Draw a 3 connected graph whose edge connectivity is 4 and minimum degree is 5. 3

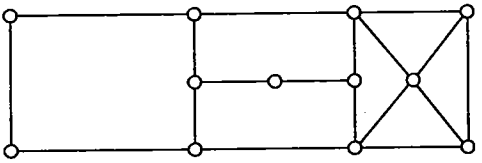
(c) Find the chromatic number of the Graph given below. If the chromatic number is k, give a k - colouring. 4



5. (a) Verify Brook's theorem for the following graph. 2

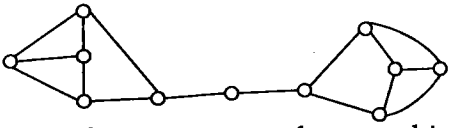


(b) State with justification whether the following graph is Hamiltonian or not. 3



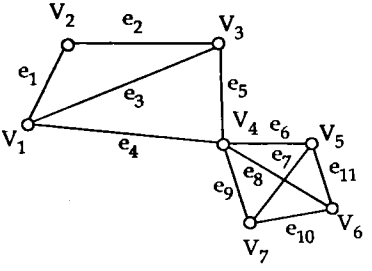
Is it Eulerian, justify your answer.

(c) Define a maximum matching and a perfect matching. Find a Maximum matching for the following graph G : 5



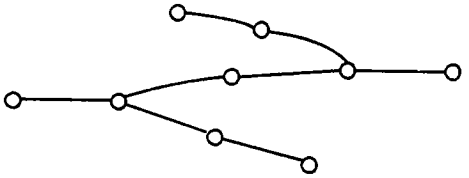
Does there exist a perfect matching for G ? Give justification.

6. (a) Find the adjacency matrix and the incidence matrix of the following graph - 4



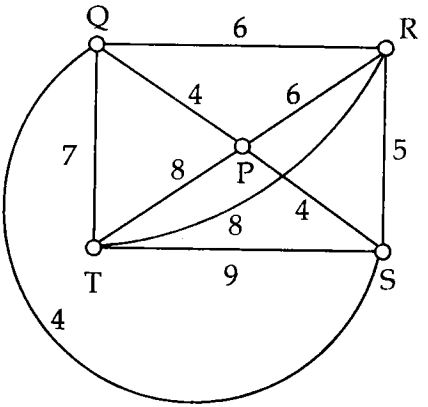
(b) How many faces will a planer graph with degree sequence 3, 3, 3, 3, 3, 3, 6 will have ? 3

(c) Find the center of the following tree : 3



7. (a) Prove that an edge e of a connected graph G is a cut edge if and only if e belongs to every spanning tree. 3

(b) Find the minimum spanning tree for the following weighted graph using Prim's algorithm. 4



(c) If G is disconnected, show that $\overline{G}$ is connected. Is the converse true ? Give justification. 3