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MMT-007

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

00823

June, 2012

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 hours *Maximum Marks : 50*

Note : *Question No. 1 is **compulsory**. Do any **four** questions out of the remaining questions 2-7. All computations may be kept to 3 decimal places. Use of calculator is **not** allowed.*

State whether the following statements are ***true*** or ***false***. Justify your answer with the help of a short proof or a counter example.

1. (a) The initial value problem **2x5=10**

$$\frac{dy}{dx} = \frac{3y - 4}{x}, \; y(0) = \frac{4}{3}$$

has an infinity of solutions.

(b) The radius of convergence of the series $1 - 2x + 3x^2 - 4x^3 +$ .... is unity.

(c) For the b v p

$$\frac{d^2y}{dx^2} - 9y = 3x,\ y(0) = y(1) = 0$$

the Green's functions $G(x, \xi)$ satisfies

$$\frac{d^2 G(x,\xi)}{dx^2} - 9\,G(x,\xi) = 3x.$$

(d) The second order Runga-Kutta method when applied to the IVP $y' = -100y$, $y(0) = 1$ produces stable results for $0 < h < 25$.

(e) In applying Milne's or Adam-Bashforth method we require four starting values of y which are calculated by means of Picards or Taylor series method only.

2. (a) The small oscillations of a certain system with two degrees of freedom are governed by the differential equations **6**

$$\frac{d^2x}{dt^2} + 3x - 2y = 0\,;\ \frac{d^2y}{dt^2} = 6x - 7y.$$

If $x = 0$, $y = 0$, $\frac{dx}{dt} = 3$, $\frac{dy}{dt} = 2$ when $t = 0$,

Obtain, using Laplace transform technique the expression for $x(t)$ and $y(t)$.

(b) Find the number of terms that are to be retained if an accuracy of 10^{-10} is required in solving the initial value problem **4**

$$\frac{dy}{dx} = x + y, \; y(0) = 1, \; x \in \,]0, 1[$$

by Taylors' series.

3. (a) Solve the following differential equation by power series method about $x = 0$ **5**

$$x^2y'' + 4xy' + (x^2 + 2)y = 0.$$

(b) Express the functions : **5**

$$f(x) = 0, \; -1 < x \leq 0$$
$$= x, \; 0 < x < 1$$

in Legendre expansion.

4. (a) Using Runge- Kutta method of fourth order find $y(0, 8)$ taking $h = 0.1$, correct to three decimal places, if : **6**

$$\frac{dy}{dx} = y - x^2, \; y(0.6) = 1.738 \,.$$

(b) Prove that $\int_0^1 x J_n(\alpha x) J_n(\beta x)\, dx = 0$ where **4**

α and β are distinct positive zeros of Bessel function $J_n(x)$, $n \geq 0$.

5. (a) Discuss stability of Euler's method for solving the differential equation $\frac{dy}{dx} = f(x, y) = \lambda y$ with initial condition $y(x_0) = y_0$. 5

(b) Show that : 5

$$\int_{-1}^{1} x P_n(x) P_{n-1}(x)\, dx = \frac{2n}{4n^2 - 1},$$

Where $P_n(x)$ is the Legendre Polynomial of order n.

6. (a) Solve the following boundry-value problem by determining the appropriate Green's function via the method of variation of parameters. Express the solution as a definite integral. 5

$$-\left(\frac{d^2y}{dx^2} + 4y\right) = f(x),\ y(1) = 0,\ y'(0) = 0$$

(b) Given $\frac{dy}{dx} = \frac{1}{2}(1 + x^2)y^2$ and $y(0)=1$, 5

$y(0.1) = 1.06, y(0.2) = 1.12,\ y(0.3) = 1.21$, evaluate $y(0.4)$ using Milnes' predictor-corrector method.

7. (a) Solve the Poissons' equation **6**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -10\left(x^2 + y^2 + 1\right) \text{ over the}$$

square mesh with sides $x=0$, $y=0$; $x=3$, $y=3$ with $u=0$ on the boundry and mesh length 1. Use five point formula.

(b) Find the Fourier cosine transform of : **4**

$$f(x) = \begin{cases} x, & 0 < x < \frac{1}{2} \\ (1-x), & \frac{1}{2} < x < 1 \\ 0, & x > 1 . \end{cases}$$