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BME-001

BACHELOR OF TECHNOLOGY IN MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

Term-End Examination

01899

June, 2012

BME-001 : ENGINEERING MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are* ***compulsory****. Use of calculator is allowed.*

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1. Answer *any five* of the following : **5x4=20**

(a) Given $f(x)=\begin{cases} a & \text{if } x \text{ integer} \\ b & \text{otherwise} \end{cases}$ where $b \neq a$.

Does $\lim_{x \to 0} f(x)$ exist. If yes, find its value.

(b) Discuss the continuity of the function $f(x)=[x]$, $x \in R$ at $x=0$. Where $[\bullet]$ is greatest integer function.

(c) Attempt any one part of the following :

(i) Find the interval in which the function $f(x)=2x^3-3x^2-36x+7$ is

(A) strictly increasing

(B) strictly decreasing

(ii) Show that of all the rectangle of given area the square has the smallest perimeter.

(d) Find the area enclosed between the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and above the line $\frac{x}{a} + \frac{y}{b} = 1$ which lies in the first quadrant.

(e) If $x+y+z=u$, $y+z=uv$, $z=uvw$ then show that $\frac{\partial (x, y, z)}{\partial (u, v, w)} = u^2 v$.

(f) Solve the differential equation $y \sin 2x \, dx - (y^2 + \cos^2 x) dy = 0$.

2. Answer *any four* of the following : **4x5=20**

(a) Find the directional derivative of $\phi = 5x^2 y - 5y^2 z + \frac{5}{2} z^2 x$ at the point (1, 1, 1) in the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$.

(b) Show that $\nabla r^n = n \, r^{n-2} \vec{r}$ and hence evaluate $\nabla \left(\frac{1}{r} \right)$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

(c) A fluid motion is given by

$$\vec{v} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$$

(i) Is this motion irrotational ?

(ii) Is the motion possible for an incompressible fluid ?

(d) Evaluate $\iint_s \vec{A} . \hat{n}\, ds$, where

$\vec{A} = (x + y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$ and s is the surface of the plane $2x + y + 2z = 6$ in the first octant.

(e) Use Gauss divergence theorem to evaluate the surface integral

$\iint_s (xdydz + ydzdx + zdxdy)$ where s is the portion of the plane $x + 2y + 3z = 6$ which lies in the first octant.

(f) Using Green's theorem, evaluate

$\int_C (x^2 y dx + x^2 dy)$ where C is the boundary described counter clockwise of the triangle with vertices (0, 0), (1, 0), (1, 1).

3. Answer *any five* of the following : **5x3=15**

(a) Employing elementary transformations, find the inverse of the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}.$$

(b) For the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$, find non - singular matrices P and Q such that PAQ is in the normal form.

(c) Determine the values of 'a' and 'b' for which the system

$$\begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b \\ 3 \\ -1 \end{bmatrix} \text{has}$$

(i) a unique solution

(ii) no solution

(iii) infinitely many solution.

(d) Show that the vectors $x_1 = (1, 2, 4)$, $x_2 = (2, -1, 3)$ $x_3 = (0, 1, 2)$ and $x_4 = (-3, 7, 2)$ are linearly dependent.

(e) Find the characteristic equation of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ and hence find the eigen - value.

(f) Solve the system by Cramer's rule :

$2x - y + 3z = 2$, $x + 3y - z = 11$,

$2x - 2y + 5z = 3$.

(g) Verify Cayley - Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

4. Answer *any three* of the following : **3x5=15**

(a) In a bolt factory, machines A, B and C manufacture 25%, 35% and 40% of the total output respectively. Of their outputs, 5%, 4% and 2% are defective bolts. A bolt is chosen at random and found to be defective. What will be the probability that the bolt came from machine A, B and C ?

(b) The probability that a man aged 60 will live to be 70 is 0.65. What is the probability that out of 10 men, now 60, at least 7 will live to be 70 ?

(c) A box contains 2 black, 4 white and 3 red balls. One ball is drawn at a time randomly from the box till all the balls are drawn from it. Find the probability that the balls drawn are in the sequence of 2 black, 4 white and 3 red.

(d) If the variance of the Poisson distribution is 2, find the probabilities for $r = 1, 2, 3, 4$ from the recurrence relation of the Poisson distribution. Also find $P(r \geq 4)$.

(e) Ten individuals are chosen at random from the population and their heights are found to be inches 63, 63, 64, 65, 66, 69, 69, 70, 70, 71. Discuss the suggestion that the mean height in the universe is 65 inches given that for 9 degree of freedom the value of student's 't' at 0.05 level of significance is 2.262.
