B.Tech. Civil (Construction Management)/ B.Tech. Civil (Water Resources Engineering) B.Tech. (Aero space Engineering)

Term-End Examination

01289

June, 2012

ET-101(A) : MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are **compulsory**. Use of calculator is allowed.*

1. Answer *any five* of the following : **5x4=20**

(a) If $f(x) = \cos x \qquad 0 \le x < \frac{\pi}{2}$

$= x - \frac{\pi}{2} \qquad \frac{\pi}{2} < x \le \pi$

$= 1 \qquad x = \frac{\pi}{2}$

Discuss the continuity of f(x) at $x = \pi/2$

(b) Find $\frac{dy}{dx}$, if $y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$

(c) Evaluate (*any one* of the following)

(i) $\lim_{x \to 0} \frac{1 - \cos x}{\sin^2 x}$

(ii) $\lim_{n \to \infty} \left(\sqrt{n^2 + n} - n\right)$

(d) Show that the normal at the point $\theta = \pi/4$ to the curve

$x = 3\cos\theta - \cos^3\theta;\ y = 3\sin\theta - \sin^3\theta$

passes through the origin.

(e) Calculate the radius and the height of a right circular cylinder of maximum volume which can be cut from a sphere of radius R.

(f) If $x = r\cos\theta,\ y = r\sin\theta,\ z = z,$

find $\dfrac{\partial(x, y, z)}{\partial(r, \theta, z)}$

2. Answer *any four* of the following : **4x4=16**

(a) Evaluate (*Any one* of the following)

(i) $\int x \sin x^2\, dx$

(ii) $\displaystyle\int \frac{(\sin^{-1}x)^3}{\sqrt{1-x^2}}\, dx$

(b) Evaluate $\displaystyle\int_{\theta}^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$

(c) Find the area bounded by :
$y^2 = 9x$ and $x^2 = 9y$

(d) Calculate $\displaystyle\int_0^{10} \frac{dx}{1+x^2}$, using Simpson's one - third rule with ten interval.

(e) Solve *(any one)*

(i) $\frac{dy}{dx} = (4x + y + 1)^2$

(ii) $\cos x \frac{dy}{dx} = y \sin x + y^3 \cos^2 x$

(f) If $z = e^{ax+by}$ and $(ax - by)$, prove that

$$b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2\,ab\,z$$

3. Answer *any four* of the following : **4x4=16**

(a) If $R = x\hat{i} + y\hat{j} + z\hat{k}$, show that

(i) $\nabla . R = 3$ (ii) $\nabla \times R = 0$

(b) Show that the following vector is solenoidal :

$$\left(-x^2 + yz\right)\hat{i} + \left(4y - z^2x\right)\hat{j} + (2xz - 4z)\hat{k}$$

(c) Find curl (curl A)

Given $A = x^2 y\hat{i} + y^2 z\hat{j} + z^2 y\hat{k}$

(d) Find the total work done in moving a particle in a force field given by

$F = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$.

(e) Show that the following vector is irrotational, and find the scalar potential

$$F = 2xy\hat{i} + (x^2 + 2yz)\hat{j} + (y^2 + 1)k$$

(f) Evaluate $\iint F.n\,ds$ where $F = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and s is the surface of the cube bounded by $x=0$, $x=1$, $y=0$, $y=1$, $z=0$, $z=1$.

4. Answer *any six* of the following : **6x3=18**

(a) Show that $\begin{vmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{vmatrix}$ is a Hermitian matrix.

(b) Find the sum and product of the eigen values of $\begin{bmatrix} 2 & 3 & -2 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$.

(c) Express A as the sum of a symmetric and a skew symmetric matrix, where

$$A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}.$$

(d) Verify that $\frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}$ is an orthogonal matrix.

(e) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

(f) Given

$$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$$

(g) Find the rank of the following matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$$

(h) Solve the following equations by Cramer's rule

$$x - y + 2z = 5$$
$$3x + y + z = 8$$
$$2x - 2y + 3z = 7$$