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MSTE–002

**POST GRADUATE DIPLOMA IN
APPLIED STATISTICS (PGDAST)**

Term-End Examination

December, 2023

MSTE-002 : INDUSTRIAL STATISTICS—II

Time : 3 Hours

Maximum Marks : 50

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **four** questions from the remaining question nos. 2 to 7.*

(iii) *Use of scientific calculator (non-programmable) is allowed.*

(iv) *Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.*

(v) *Symbols have their usual meanings.*

1. State whether the following statements are True or False. Give reasons in support of your answer :

$$5 \times 2 = 10$$

(a) Show that the feasible solution $x_1 = 1, x_2 = 0, x_3 = 1$ and $z = 6$ to the system of equations :

P. T. O.

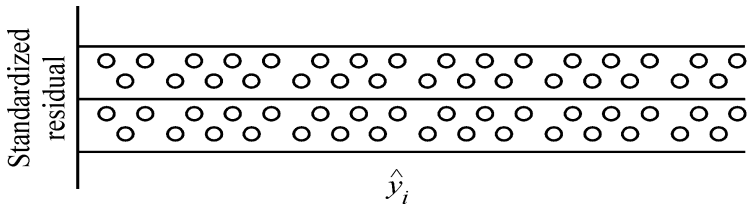
$$x_1 + x_2 + x_3 = 2$$

$$x_1 - x_2 + x_3 = 2$$

$$x_i \geq 0, i = 1, 2, 3$$

which minimizes $z = 2x_1 + 3x_2 + 4x_3$ is not basic.

- (b) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time, follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes. Then the mean arrival rate and the mean service rate are same.
- (c) If a researcher obtained a residual plot as follows :



then we can say that there exist heteroscedasticity in the data.

- (d) A series of 10 consecutive yields (y) from a chemical process, gives the estimated value of :

$$\Sigma y_t = 510, \Sigma y_t^2 = 27906,$$

$$\Sigma (y_t - \bar{y})(y_{t+1} - \bar{y}) = -1497$$

Then the value of autocorrelation coefficient r_1 is -0.99 .

- (e) In transportation problem, an optimality test can be applied when it contains exactly $m + n$ allocations.

2. Find the solution of the following L. P. P. : 10
Max. :

$$z = 3x_1 + 2x_2 + 5x_3$$

Subject to :

$$x_1 + 2x_2 + x_3 \leq 430$$

$$3x_1 + 2x_3 \leq 460$$

$$x_1 + 4x_2 \leq 420$$

and $x_1, x_2, x_3 \geq 0$.

3. (a) Solve the following transportation problem :

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	D ₁	D ₂	D ₃	D ₄	a_i
O ₁	5	3	6	2	19
O ₂	4	7	9	1	37
O ₃	3	4	7	5	34
b_j	16	18	31	25	

- (b) Solve the following assignment problem : 5

	1	2	3	4
A	10	12	19	11
B	5	10	7	8
C	12	14	13	11
D	8	15	11	9

which minimizes the total cost of the project.

4. (a) There are five jobs, each of which must go through the two machines A and B in the order AB. If the following tables gives the processing times of the two machines :

Job	Processing time (hours)				
	1	2	3	4	5
Time for A	5	1	9	3	10
Time for B	2	6	7	8	4

then determine the sequence for five jobs that will minimise the elapsed time T. Also, calculate the total idle time for machines in this period. 5

- (b) Customers arrive at a window in a bank, according to a Poisson distribution with 10 per hour. Service time per customer is exponential with mean 5 minutes. The space in front of the window including that

for the serviced customers can accommodate a maximum of three customers, other customers can wait outside this space. 5

- (i) What is the probability that an arriving customer can go directly to the space in front of the window ?
- (ii) What is the probability that an arriving customer will have to wait outside the indicated space ?
5. A study was conducted to find the effect of temperature (X) on the yield of a chemical process (Y). The following data were collected :

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X	Y
-5	-1
-4	5
-3	4
-2	7
-1	10
0	8
1	9
2	13
3	14
4	13
5	18

- (i) Assuming a model $Y = a + bX + e$, what are the least squares estimates of a and b ?
- (ii) Calculate the variances of the estimated regression coefficients.
- (iii) Test the hypothesis that the temperature (X) has no effect on the yield (Y) of the chemical process, i.e. $H_0 : b = 0$, $H_1 : b \neq 0$ at $\alpha = 0.05$.
6. (a) Describe the properties (in brief) of the estimates of the parameters a and b for the regression model $Y = a + bX + e$. 5
- (b) Fit a straight line trend for the data of annual profit of a company given below : 5

Year	Profit (in crores)
2003	93
2004	102.8
2005	126.7
2006	103.5
2007	105.7
2008	133.2
2009	156.7
2010	175.7
2011	161.6

7. (a) Define autoregressive (AR) process.
Consider the following AR (2) process : 8

$$X_t = X_{t-1} - 0.3X_{t-2} + a_t$$

Answer the following :

- (i) Check whether the series is stationary or not.
 - (ii) Obtain ρ_k for $k = 1, 2, 3, 4$.
 - (iii) Plot the correlogram.
- (b) Differentiate between the ARMA and ARIMA models. 2