No. of Printed Pages : 6 **MMTE–005**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M. SC. (MACS)

Term-End Examination

December, 2023

MMTE–005 : CODING THEORY

Time : 2 Hours *Maximum Marks : 50*

Note : (*i*) *There are* ***six*** *questions in this paper.*

(*ii*) *The* ***sixth*** *question is compulsory.*

(*iii*) *Do any* ***four*** *questions from question* ***one*** *to question* ***five***.

(iv) Show all the relevant steps. Do the rough work at the bottom or at the side of the page only.

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1. (a) When do we say that the parity check matrix of a $[n,k]$ linear code is in standard from. Check whether the parity matrix

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

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of a linear code **C** is in standard form or not. Also, determine the length and dimension of **C** ? 3

(b) Define the Hamming distance between two codewords. Find the Hamming distance between the two codewords 000110 and 110101. 2

(c) What is a repetition code ? If, in a repetition code in which the message of length is sent thrice, the codeword 110 111 110 is received, decode the message assuming there is at most one error. 2

(d) Let **C** be a binary cyclic of length nine with generator polynomial $x^6 + x^3 + 1$. Find the generator matrix and the parity check matrix of the code **C**. 3

2. (a) Let $n \in \mathbf{N}, q$ be a power of a prime and $0 \leq s < n$. Define the q-cyclotomic coset of s modulo n. Find the 8-cyclotomic set of 1 modulo 19. 3

(b) Does there exist a quadratic residue code of length 17 over $\mathbf{F}_{11}$? Give reasons for your answer. 2

(c) Find the gcd of
$x^5+2x^3+2x^2+x+1, x^3+x+1 \in \mathbf{F}_3[x]$. 2

(d) Let $\mathbf{C}_1$ be the [4,3,2] binary linear code generated by $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ and let $\mathbf{C}_2$ be the [4,1,4]- binary linear code generated by [1111]. Let **C** be the code obtained through using ($\mathbf{u}|\mathbf{u}$ + v) construction on the codes $\mathbf{C}_1$ and $\mathbf{C}_2$. Find the generator matrix of **C**. Also, give the length and the dimension of **C**. 3

3. (a) Prove that the integers modulo n do not form a field if n is not a prime. 2

(b) The systematic generator matrix for a [6,3] linear block code is $\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$. Compute the standard array for syndrome decoding. 5

(c) Check whether the polynomical $x^3+x+1 \in \mathbf{F}_5$ is primitive. You are given that $x^{30} \equiv x^2+1 \pmod{x^3+x+1}$. You may assume that the polynomical is irreducible. 3

4. (a) Find a generator polynomial for a [13,10]-BCH code. Use $x^3 + 2x + 2 \in \mathbf{F}_3[x]$ as the primitive polynomial for F_{27}, and Table 1. 5

(b) Find the weight distribution of the binary code generated by

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Find the weight enumerator polynomial of the code. Also, find the weight enumerator poly-nomial of the dual code. 5

5. (a) Let **C** be the [7,4,2] binary code with the following parity check matrix :

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

i	α^i
1	α
2	α^2
3	$\alpha + 2$
4	$\alpha^2 + 2\alpha$
5	$2\alpha^2 + \alpha + 2$
6	$\alpha^2 + \alpha + 1$
7	$\alpha^2 + 2\alpha + 2$
8	$2\alpha^2 + 2$

9	$\alpha + 1$
10	$\alpha^2 + \alpha$
11	$\alpha^2 + \alpha + 2$
12	$\alpha^2 + 2$
13	2
14	2α
15	$2\alpha^2$
16	$2\alpha^2 + 1$
17	$2\alpha^2 + \alpha$
18	$\alpha^2 + 2\alpha + 1$
19	$2\alpha^2 + 2\alpha + 2$
20	$2\alpha^2 + \alpha + 1$
21	$\alpha^2 + 1$
22	$2\alpha + 2$
23	$2\alpha^2 + 2\alpha$
24	$2\alpha^2 + 2\alpha + 1$
25	$2\alpha^2 + 1$

Table 1 : Powers of $a \in \mathbf{F}_{27}$ where $a^3 + 2a + 1 = 0$

(i) Give the Tanner graph for this code. 3

(ii) List all the codeword's of **C** and hence find its minimum distance. 2

(b) List all the code words of the code **C** over $\mathbf{Z}_4$ generated by

$$\begin{bmatrix} 1 & 2 & 3 & 0 & 1 \\ 2 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Also find the Lee weight distribution of the code. 5

6. Which of the following statements are true and which are false ? Justify your answer with short proof or a counter example. 10

(a) If F is a field and the polynomial $p(x) \in F[x]$ has no roots in F, then $p(x)$ is irreducible over F.

(b) The code with generator matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

has a unique codeword of weight three.

(a) A quadratic code of length seven exists over $\mathbf{F}_3$.

(b) The parity check matrix of a turbo code can be the identify matrix.

(e) Every perfect code is self dual.