No. of Printed Pages : 4

MMTE–001

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. SC. (MACS)]

Term-End Examination

December, 2023

MMTE-001 : GRAPH THEORY

Time : 2 Hours *Maximum Marks : 50*

Weightage : 50%

Note : *Question No.* ***1*** *is compulsory. Answer any* ***four*** *questions from Question Nos.* ***2*** *to* ***7****. Use of calculators is not allowed.*

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1. State whether the following statements are true *or* false. Justify your answers with a short proof or a counter-example : 10

   (i) There exists a graph of order 7 and size 25.

   (ii) diam $(K_{m,n}) = 2$ for all $m \geq 1$, $n \geq 1$.

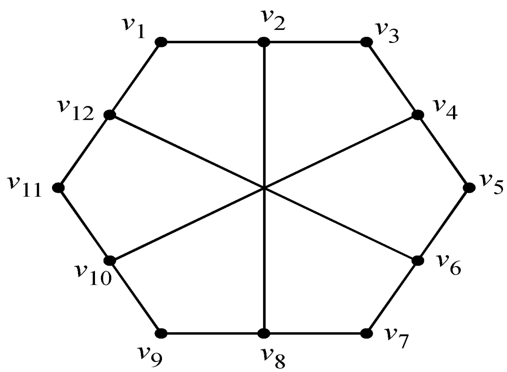
   (iii) If G is a graph with no cycles, then every edge of G is a cut-edge.

   (iv) Every Eulerian graph has a perfect matching.

(v) If G is a graph in which every vertex lies in a cycle, then G is 2-connected.

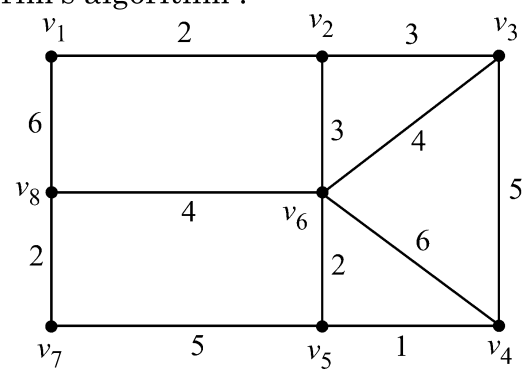
2. (a) Show that if G is a simple graph with diam (G) > 3, then diam ($\overline{G}$) ≤ 3. 6

(b) Check whether the graph given below is bipartite or not : 4

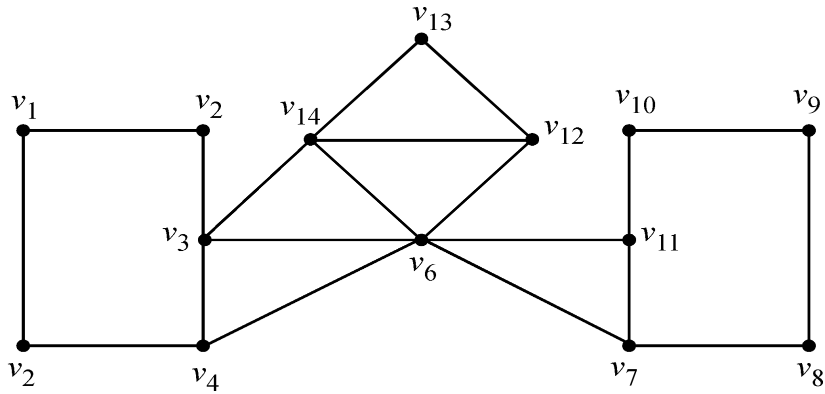


3. (a) Prove that every tree on n vertices has exactly $n - 1$ edges. 4

(b) Find a minimum weight spanning tree in the weighted graph given below, using the Prim's algorithm : 6



4. (a) Find a perfect matching in the graph given below, if it exists. Otherwise, justify why no such matching exists : 6



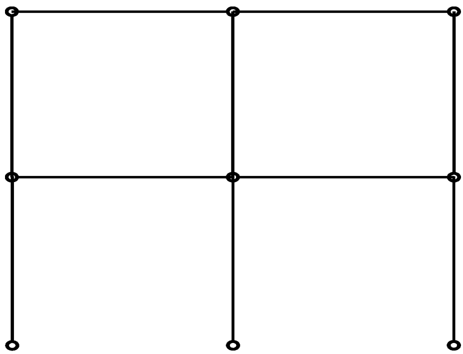
(b) Does there exist a 3-regular graph with a cut-vertex ? If yes, construct such a graph. 4

5. (a) Show that a graph is 2-chromatic iff it is bipartite. 3

(b) If G is a planar graph with at least three vertices, then show that G has at most $3n - 6$ edges, where n is the order of G. 5

(c) Let G be a graph obtained by detecting an edge from K_5. Is G planar ? If yes, give a plane drawing of G. 2

6. (a) Draw the dual of the following plane graph : 3



(b) Let G be a graph with order $n \geq 3$. If $\delta(G) \geq \frac{n}{2}$, then show that G is Hamiltonian. 7

7. (a) Check whether (7, 5, 3, 3, 2, 2, 1, 1) is a graphic sequence or not. If yes, construct a graph with this degree sequence. 7

(b) Show that for every odd cycle the chromatic number and edge-chromatic number are the same. 3