No. of Printed Pages : 4

MMT–009

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. SC. (MACS)]

Term-End Examination

December, 2023

MMT-009 : MATHEMATICAL MODELLING

Time : $1\frac{1}{2}$ *Hours* *Maximum Marx : 25*

Weightage : 70%

Note : *(i) Attempt any* ***five*** *questions.*

(ii) Use of scientific non-programmable calculator is allowed.

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1. (a) List the *two* essentials and *two* non-essentials in the problem to develop a model to help the insurance company decide the premium it should charge for different risks to ensure economic viability and maximise its profits. 2

(b) Calculate the expected return and risk of a security given the following information : 3

Probabilities (P_{ij})	**Returns, R_j**
0.15	0.20
0.20	0.16
0.40	0.12
0.10	0.05
0.15	– 0.05

2. Find a linear demand equation that best fits the following data, and use it to predict annual sales of homes priced at ₹ 14,00,000 : 5

***x* = Price (lakhs of ₹)**	***y* = Sales of new homes this year**
16	126
18	103
20	82
22	75
24	82
26	40
28	20

3. Do the stability analysis of the following model which is formulated to study the effect of toxicant on a competing species : 5

$$\frac{dN_1}{dt} = r_1 N_1 - \alpha_1 N_1 N_2 - d_1 C_0 N_1$$

$$\frac{dN_2}{dt} = r_2 N_2 - \alpha_2 N_1 N_2$$

$$\frac{dC_0}{dt} = k_1 P - g_1 C_0 - m_1 C_0$$

$$N_1(0) \geq 0,\ N_2(0) \geq 0, C_0(0) = 0$$

4. (a) Define the following terms : 2

   (i) Variational matrix

   (ii) Hurwitz's criteria

   (b) A spherical tumour is occupying the space in an organ with outer and inner radii 0.2×10^{-6} mm and 0.001×10^{-9} mm, respectively for tumour and cancer cells. Find the density of cancer cells in the vicinity of the surface. 3

5. A tax consulting firm has 3 counters to receive people who have problems concerning their incomes, wealth and taxes. On the average 48 persons arrive in an 8-hour day. Each tax adviser spends 15 min. on an average on an

arrival. If the arrivals are Poissonly distributed and service times are according to exponential distribution, find : 5

(i) average number of customers in the system;

(ii) average number of customers waiting to be served;

(iii) average time a customer spends in the system.

6. Discuss the linear stability of the following discrete time population models : 5

$$x_{n+1} = \frac{rx_n}{x_n + \mathrm{A}}$$