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# MMT–005

## M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

## [M. Sc. (MACS)]

## Term-End Examination

## December, 2023

### MMT-005 : COMPLEX ANALYSIS

*Time : 1 $\frac{1}{2}$ Hours* *Maximum Marks : 25*

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***Note :*** (*i*) *Question No.* ***1*** *is compulsory.*

(*ii*) *Attempt any* ***three*** *questions from Question Nos.* ***2*** *to* ***5****.*

(*iii*) *Use of calculator is not allowed.*

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1. State, giving reasons whether the following statements are True *or* False : 5×2=10

   (a) $\{z : 2\,\mathrm{Re}\, z + 3\,\mathrm{Im}\, z \leq 2\}$ is both open and closed.

   (b) $\lim_{z \to -i} \frac{z^4 - 1}{z + i} = 4i$

   (c) $w = z + \frac{1}{z}$ is conformal for all $z \in \mathbf{C}/\{0\}$

(d) The function $f(z) = |z|^2$ is analytic on **C**.

(e) $\int_C \frac{dz}{z-1} = 0$, where C : $|z| = \frac{1}{2}$.

2. (a) Find the Laurent series expansion of the function $f(z) = \frac{1}{z(z-1)}$ in the regions : 3

(i) $0 < |z-1| < 1$

(ii) $|z-1| > 1$

(b) Find an analytic function whose real part is $u(x, y) = e^{-y} \sin x$. 2

3. (a) Find the images of the points 0, $1 + i$, $i$ and $\infty$ under the linear fraction : 3

$$f(z) = \frac{2z+1}{z-i}$$

(b) Show that the zeroes of an analytic function are isolated. 2

4. (a) Find the maximum modulus of $f(z) = 2z + 5i$ on the closed circular region defined by $|z| \leq 2$. 2

(b) Evaluate $\int_C \frac{\cos z}{z^3 + z} dz$, where C is the circle $|z| < = 2$. 3

5. Evaluate :

$$\int_0^{\pi} \frac{(1 + 2\cos\theta)}{5 + 4\cos\theta} d\theta$$

using Cauchy residue theorem. 5