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M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. Sc. (MACS)]

Term-End Examination

December, 2023

MMT-003 : ALGEBRA

Time : 2 Hours *Maximum Marks : 50*

Note : *Question **No. 1** is **compulsory**. Answer any **four** questions from **Q. Nos. 2** to **6**. Calculators are not allowed. Show all the steps involved.*

1. Which of the following statements are true and which are false ? Justify your answer with a short proof or a counter-example : 10

   (i) The rings $\frac{\mathbf{R}[x]}{\langle x^2+1\rangle}$ and $\mathbf{R} \times \mathbf{R}$ are isomorphic.

   (ii) If L and K are finite extensions of a field $F \subseteq \mathbf{C}$, then [KL : F] = [L : F] [K : F].

P. T. O.

(iii) If $r, s \in \mathbf{N}$, $r, s > 1$, then S_{rs} has an element of order $r + s$.

(iv) Every free abelian group is a free group.

(v) For any finite group G and $g \in G$, $o(g) = |Z(g)|$.

2. (a) Must a group of order 30 contain an element of order 15 ? Give reasons for your answer. 8

(b) If $\mathbf{F}_{2^4}$ a field extension of $\mathbf{F}_{2^3}$? Justify your answer. 2

3. (a) Let K be a Galois extension of a field F, with Galois group G (K/F) isomorphic to S_3. How many fields L will be there such that $F \underset{+}{\subset} L \underset{+}{\subset} K$? How many such L will be normal extensions of F ? Justify your answers. 4

(b) Show that the ring $\mathbf{Z}[\sqrt{-11}]$ is not a Euclidean domain. 6

4. (a) Determine all the possible abelian groups, upto isomorphism, of order 1400. 4

(b) Consider an action of the quaternion group on a set with 11 elements. Show that this action has at least one fixed point. 3

(c) Compute the Legendre symbol $\left(\frac{63}{41}\right)$. 3

5. (a) Show that $SL_2(\mathbf{Z}) \cap SO_2(\mathbf{R})$ is a cyclic group of order 4. 3

(b) Find the splitting field of $x^{12} + T$ in $\mathbf{Z}_5[x]$, over $\mathbf{Z}_5$. Also find its degree over $\mathbf{Z}_5$. 7

6. (a) Give an example, with justification, of a ring R and a prime ideal I of R which is not maximal in R. 2

(b) Write $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & -1 \\ 0 & 2 & 1 \end{bmatrix}$ as a product of elements of $O_3(\mathbf{R})$ and $B_3(\mathbf{R})$. 5

(c) Find a permutation group isomorphic to $\mathbf{Z}_4$.

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