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MMT–002

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. Sc. (MACS)]

Term-End Examination

December, 2023

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ Hours *Maximum Marks : 25*

Note : *Question No.* ***5*** *is compulsory. Answer any* ***three*** *questions from Q. Nos.* ***1*** *to* ***4****. Calculators are not allowed.*

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1. (a) Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be a linear transformation defined by :

$$T(x, y, z) = (3x + 2y - 4z,\ x - 5y + 3z)$$

Find the matrix of T relative to the bases {(1, 1, 1), (1, 1, 0), (1, 0, 0)} or $\mathbf{R}^3$ and {(1, 0), (1, 1)} of $\mathbf{R}^2$. 2

P. T. O.

(b) Find the spectral decomposition of : 3

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

2. (a) Find the QR decomposition of A and hence find a least squares solution of the system $Ax = b$, where : 3

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \text{ and } b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}.$$

(b) Write all possible Jordan canonical forms of a 5×5 matrix having :

$$(t-2)^2 (t-3)(t-4)$$

as the minimal polynomial. 2

3. Find the singular value decomposition of : 5

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$$

4. (a) Prove that if N is a non-zero nilpotent operator, then N is not diagonlizable. 2

(b) Check whether or not $A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{bmatrix}$ is unitarily diagonalizable. If it is, find a unitary matrix U such that U* AU is diagonal. If A is not unitarily diagonalizable, obtain the Schur decomposition of A. 3

5. Which of the following statements are true and which false ? Justify your answer with a short proof or a counter-example, whichever is appropriate : 2×5=10

(i) If two matrices have the same characteristic polynomial, they are similar.

(ii) An invertiable matrix must be positive definite.

(iii) A and AA^t have the same rank for any matrix A.

(iv) For every matrix S in $M_n(\mathbf{R})$, there is an $n \times n$ orthogonal matrix O such that $O^t SO \in M_n(\mathbf{R})$ is upper triangular.

(v) If N is a nilpotent matrix, then e^N is also nilpotent.