MCS–013

M. C. A. (REVISED)/B. C. A. (REVISED) (MCA/BCA)
Term-End Examination
December, 2023
MCS–013 : DISCRETE MATHEMATICS

Time : 2 Hours   *Maximum Marks : 50*
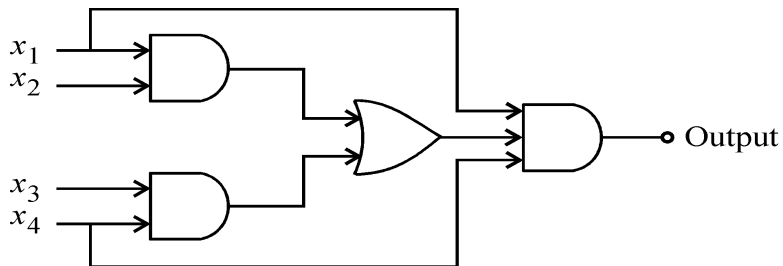
Note : *Question No. **1** is compulsory. Attempt any **three** questions from the rest.*

1. (a) Using truth table, show that :   2

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

   (b) Prove that $\sqrt{2}$ is irrational.   2

   (c) Find the Boolean expression for the output for the following circuit :   2

(d) Make Venn diagram for the following set of expressions :   2

(i) $\bar{A}$

(ii) $A \Delta B$ (Symmetric Difference)

(e) Find the domain for which the function $f(x) = 3x^2 - 1$ and $g(x) = 1 - 5x$ are equal.

2

(f) In how many ways can a student choose 8 questions out of 10 questions in an exam ?   2

(g) Prove the following :   2

$$\sim \left(\exists_x P(x) \right) \equiv \forall_x \left(\sim P(x) \right)$$

(h) Let $f(x) = \dfrac{1}{x}$ and $g(x) = x^3 + 2$, where $x \in R$. Find $(f + g)\,(x)$ and $(fg)(x)$.   2

(i) Use mathematical induction to prove :   2

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

(j) How many distinct three-letter words can be formed from the letters of the word "MUST" ?   2

2. (a) Let $A = R - \{3\}$ and $B = R - \{1\} : f : A \to B$, defined by $f(x) = \dfrac{x-2}{x-3}$. Find f^{-1}.   5

   (b) Write the contrapositive and converse of the following statement :   2

   "if 2 + 2 = 5, then I am a Lion".

   (c) What is dual in a Boolean expression ? Explain the principle of duality with the help of an example.   3

3. (a) Compare predicate and preposition logic. Give De Morgan's laws for both. Also, give suitable example for both.   5

   (b) Explain Pigeon hole principle with suitable example.   5

4. (a) Let :

$$A = \{a,b,c,d\}$$

$$B = \{1,2,3\}$$

   and $R = \{(a,2),(b,1),(c,2),(d,1)\}.$

   Is R a function ? Explain.   2

(b) Show that in any group of 30 people, we can always find 5 people who were born on the same day of the week. 　3

(c) Write short notes on any *two* of the following : 　5

(i) Modus-Ponens

(ii) Disjunctive syllogism

(iii) Contrapositive

5. (a) Show that for integer greater than zero : 　3

$$2^n >= n+1$$

(b) Let A and B be two mutually exclusive events such that $p(A) = 0.6$ and $p(B) = 0.3$. What is the probability that : 2

(i) A does not occur ?

(ii) A and B both occur simultaneously ?

(c) Reduce the following expression to the simpler form : 　2

$$F(a,b,c) = (a' \wedge b' \wedge c') \vee (a' \wedge b' \wedge c)$$

$$\vee (a \wedge b \wedge c')$$

(d) Prove that : 　3

$$^nC_r + {}^nC_{r-1} = {}^{n+1}C_r \left(0 \le r \le n\right)$$