POST GRADUATE DIPLOMA IN APPLIED STATISTICS (PGDAST)

Term-End Examination

December, 2022

MST-003 : PROBABILITY THEORY

Time : 3 hours *Maximum Marks : 50*

Note :

(i) Question no. ***1*** *is* ***compulsory****.*

(ii) Attempt any ***four*** *questions from the remaining Questions No. 2 to 7.*

(iii) Use of scientific (non-programmable) calculator is allowed.

(iv) Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.

(v) Symbols have their usual meanings.

1. State whether the following statements are *True* or *False*. Give reasons in support of your answers. *5×2=10*

(a) The only condition for applying classical definition of probability is that outcomes should be equally likely.

(b) If two events A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$.

(c) For the following probability distribution

X	– 2	– 1	0	1	2
p(X)	0·15	0·30	0	0·30	0·25

$E(2X + 3)$ will be 3·4.

(d) If a random variable X follows a Poisson distribution such that

$$P[X = 1] = 2P[X = 2]$$

then variance of X will be 1.

(e) If telephone calls arrive at a switchboard at an average rate of 2 per minute, then the waiting time until the 4^{th} call arrives will follow the exponential distribution.

2. (a) The probabilities that a person arriving at a petrol pump will fill petrol or CNG or both are 0·12, 0·29 and 0·07, respectively.

(i) What is the probability, that a person arriving at this pump will fill neither petrol nor CNG ?

(ii) Find the probability, that a person who fills petrol, will also fill CNG. Assume that all persons arriving at the petrol pump have a vehicle with petrol and CNG. *5*

(b) By examining the body temperature and other symptoms, the probability that corona is detected when a person is actually suffering is 0·99. The probability of the doctor diagnosing incorrectly that a person has corona on the basis of body temperature and other symptoms is 0·001. In a certain city, 1 in 1000 persons suffers from corona. A person selected at random is diagnosed to have corona. What is the probability that he actually has corona ? *5*

3. (a) If probability distribution of random variable X is as follows :

X	0	1	2	3
p(x)	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{5}{10}$	$\frac{1}{10}$

then find the probability distribution of $Y = X^2 + 2X$. *4*

(b) Let X and Y be two random variables having joint pdf

$$f(x, y) = \begin{cases} kxy, & \text{for } 0 < x < 4, \quad 1 < y < 5 \\ 0, & \text{otherwise} \end{cases}$$

(i) For f(x, y) to be a joint density function, what must be the value of k ?

(ii) Find marginal density functions of X and Y.

(iii) Check for independence of X and Y. *2+3+1*

4. (a) The probability that a student who enrolled in a bachelor programme of IGNOU will graduate is 0·4. Assume that all students who enrolled in bachelor programmes are equally likely to pass it. Determine the probability that out of 5 students.

(i) none,

(ii) one,

(iii) at least one, and

(iv) all

will graduate. *5*

(b) A company produces and ships 16 personal computers knowing that four of them have defective wiring. The company that purchased the computers is going to thoroughly test three of the computers. What is the probability that the purchasing company will find the following ? *5*

(i) No defective computer

(ii) Exactly three defective computers

(iii) Two or more defective computers

(iv) One or no defective computer

5. (a) The life-time (in hours) of a certain electrical equipment follows the normal distribution with mean = 80 and SD = 16. What is the probability that the equipment lasts :

(i) at least 100 hours,

(ii) between 80 to 100 hours,

(iii) at most 110 hours ? *6*

(b) Determine the constant k such that the function $f(x) = k\ x^{1/2}\ (1-x)^{1/2}$ is a beta distribution of first kind. Also find its mean and variance. *4*

6. (a) The magnitude of earthquakes recorded in a region follows an exponential distribution, with a mean of 2·4 (as measured on the Richter scale). Find the probability that an earthquake striking this region will have its magnitude

(i) exceeding 3·0, and

(ii) between 2·0 and 3·0. *5*

(b) An oil company conducts a geological study which indicates that an exploratory oil well should have a 20% chance of striking oil.

(i) What is the probability that the first strike comes on the third well drilled ?

(ii) What is the probability that the third strike comes on the seventh well drilled ? *5*

7. (a) On the basis of past experience, the weather forecasts of a station is 1200 times correct, out of 2000 consecutive days. A day is selected at random. Find the probability that :

(i) weather forecast on this day is correct.

(ii) weather forecast on this day is false. *2*

(b) A committee of four has to be formed from 3 economists, 4 engineers, 2 statisticians and 1 doctor. What is the probability that

(i) each of four professions is represented in the committee ?

(ii) the committee consists of one statistician and at least one economist ? *3*

(c) A player tosses two fair coins. He wins ₹ 50 if 2 heads occur, ₹ 20 if 1 head occurs and loses ₹ 10 if no head occurs.

(i) Find the probability distribution of his profit.

(ii) Find his expected gain.

(iii) Is the game fair ? *5*