No. of Printed Pages : 4 **MMTE-002**

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. Sc. (MACS)]

Term-End Examination

December, 2022

MMTE-002 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Answer any **four** out of questions **1** to **5**.*

*(ii) Question No. **6** is compulsory.*

1. (a) Explain what is an algorithm with the help of an example. 2

(b) Sort the following sequence of numbers using INSERTION-SORT showing all the steps : 3

8, 2, 4, 3, 15.

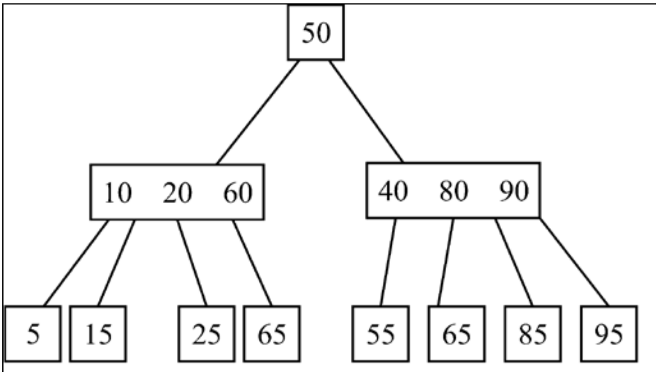
(c) Build a max heap by successive insertion of the following sequence of data : 5

5, 3, 17, 10, 84, 19.

2. (a) Illustrate the counting sort algorithm using the following array : 5

$\{3, 5, 2, 3, 4, 1, 2, 1, 4, 3\}$

(b) State the properties of a B-tree. Verify whether the following is a B-tree : 5

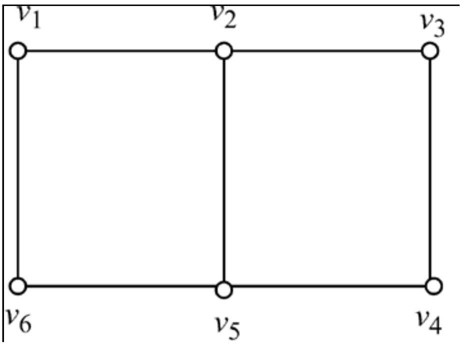


3. (a) Give in pseudo code the MERGE procedure of MERGE SORT algorithm. Explain it with the following arrays : 6

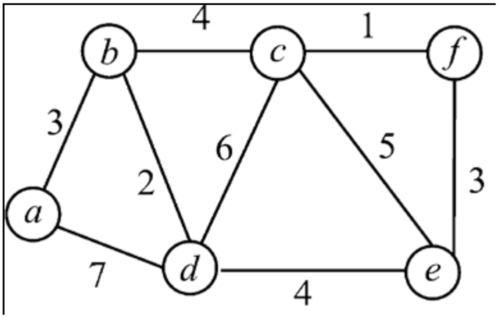
2	4	6	8

1	3	5	9

(b) Illustrate the depth-first algorithm using the following graph starting from v_i : 4



4. (a) Find a minimal spanning tree of the following graph using Prim's algorithm : 5



(b) Find the longest common substring of the following strings using Dynamic programming : 5

X = {D, C, B, C, A, D, C}

Y = {C, A, B, D, C, D}

5. (a) Show the comparisons that the naive string matching algorithm makes for the pattern : 4

P = *aaab*,

T = $\bar{a}b\ aaa\ babaaab$

(b) Define a flow network. Show that, if f_1 and f_2 are flows, $\alpha f_1 + \beta f_2$ is also a flow, where $\alpha + \beta = 1,\ 0 \leq \alpha, \beta \leq 1.$ 3

(c) Let $f(n) = 2^3 + 4^3 + 6^3 + \ldots\ldots + (2n)^3$. Find a function $g(n)$ such that $f(n) = \mathrm{H}(g(n))$. 3

6. Which of the following statement are true and which are false ? Justify your answer with a short proof or a counter example : 5×2=10

   (a) Any array in ascending order is a min heap.

   (b) The fractional Knapsack problem can be solved using a dynamic programming based strategy.

   (c) The number of keys in a B-tree of minimum degree d is at most $\dfrac{\left((2t-1)^{d+1}-1\right)}{2^d}$.

   (d) The congruence $a_n \equiv b \,(\text{mod } n)$ has at least one solution for any natural number a, b and n.

   (e) For any weighted graph, there is a unique minimal spanning tree.