No. of Printed Pages : 4 **MMTE-001**

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. Sc. (MACS)]

Term-End Examination

December, 2022

MMTE-001 : GRAPH THEORY

Time : 2 Hours *Maximum Marks : 50*

Note : *Question No. **1** is **compulsory**. Answer any **four** questions from Question Nos. **2** to **7**. Use of calculator is not allowed.*

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1. State whether the following statements are true *or* false. Justify your answers with a short proof or a counter-example : 5×2=10

   (i) There exists an n-vertex disconnected graph with minimum degree 1 and maximum degree $n-2$.

(ii) If G is a graph with diameter 2, then $N(u) \cap N(v) \neq \phi$ for every pair of non-adjacent vertices u and v of G.

(iii) The Petersen graph is semi-Eulerian.

(iv) Every 3-chromatic graph contains an odd cycle.

(v) Every maximal matching is a maximum matching.

2. (a) State and prove the Handshaking Lemma. 3

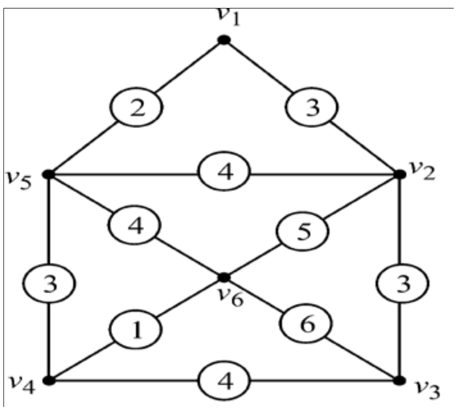
(b) Find the girth of the Petersen graph. 4

(c) Check whether there exists a tree with degree sequence : 3

$$(8, 6, 6, 6, 3, 3, 3, 1, 1, 1)$$

3. (b) Let G be a graph with diam (G) $\geq$ 3. Show that diam $(\overline{G}) \leq 3$. 5

(a) Find a minimum-weight spanning tree in the following graph, using Kruskal's algorithm : 5



4. (a) Give an example of a graph that is Hamiltonian, but not Eulerian. Justify your choice of example. 3

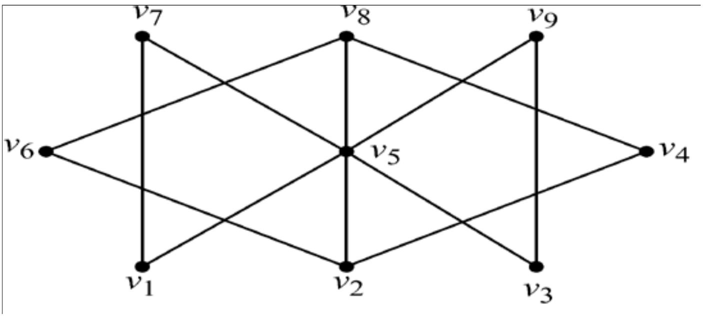
(b) For every graph G, $\chi(G) \geq \omega(G)$. True or false ? Justify. 2

(c) Prove that every k-critical graph has minimum degree at least $k - 1$. 5

5. (a) Define a planar graph. Give an example of a planar graph whose complement is also planar. 4

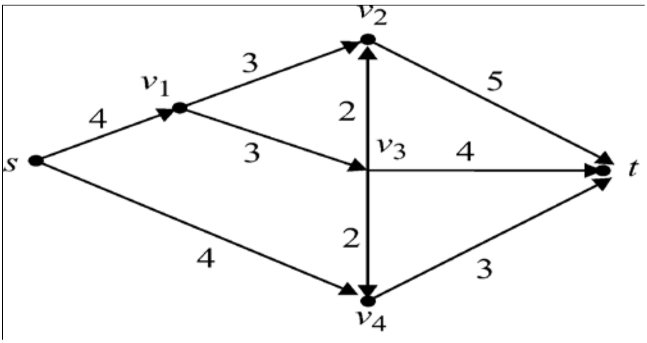
(b) Using Wagner's theorem, show that the Petersen graph is non-planar. 6

6. (a) Compute $\alpha(G), \beta(G)$ and $\alpha'(G)$, where G is the graph given below : 6



(b) Give an example of a graph G, with $\kappa(G) < \kappa'(G) < \delta(G)$. Justify your choice of example. 4

7. (a) Define a flow on the following network with value 7.



Does there exist a flow with value 8 on this network ? Why ? 6

(b) Let G be a connected graph with blocks $B_1, B_2, \ldots\ldots\ldots, B_k$. Show that : 4

$$n(G) = \sum_{i=1}^{k} n(B_i) - k + 1$$